Frequency down-conversion through Bose condensation of light

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We propose an experimental setup allowing one to convert an input light of wavelengths about 1–2 μm into an output light of a lower frequency. The basic principle of operation relies on the nonlinear optical properties exhibited by a microcavity filled with a photonic bandgap material. The light inside this material behaves like an interacting Bose gas susceptible to reach thermal equilibrium and create a Bose-Einstein condensate. Theoretical estimations show that using a fiber grating, a conversion of 1 μm into 1.5 μm is achieved with an input pulse of about 1 ns, and characterized by a peak power of 10^3 W.

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I. INTRODUCTION

The recent discovery of the Bose-Einstein condensate in atomic gas demonstrated, experimentally, the existence of a macroscopic state where a majority of particles are in the same quantum state of energy [1]. This new property is of considerable interest since it offers the possibility of creating an atom laser. By analogy with the laser, the particles move collectively to form a coherent wave. The success of this discovery is due, in part, to the ability of atoms to cool evaporatively in a trap. By eliminating the hottest atoms, some cold atoms are heated so that they get their energy from the other cold ones, ensuring a thermal equilibrium. The net result is a cooling, leading to a macroscopic population in the lowest ground state.

One question arises then: If such a thermal relaxation process is successful for atoms, why not use it for photons? Suppose that a photon gas has an initial nonequilibrium spectrum of frequency in the optical domain. If it evolves in a nonlinear medium to allow strong enough collision interactions between photons, the thermal equilibrum spectral distribution can be reached.

Based on simple dimensional analysis, we show in this paper that thermal relaxation of optical intense light is indeed possible, provided the following conditions are satisfied: (1) the light evolves in a glass medium, having a sufficiently low absorption, in such a way as to enable one to observe strong enough nonlinear interactions; (2) the glass is inside a high finesse cavity in order to confine the photons during a time greater than the relaxation time, towards thermal equilibrium; and (3) high intensity light is required to stimulate the collision process by means of Bose enhancement. A similar proposal, known as polariton laser, has been reported, where instead of direct photon-photon scattering, a particular kind of nonlinearity coming from the exciton-photon coupling and exciton-exciton scattering is considered [2].

Furthermore, in order to realize the (quasi-)condensation of light, the photon must acquire an effective mass. A two-dimensional (2D) massive Bose gas is created in a cavity, where only the fundamental mode is considered among longitudinal modes [3]. Concerning the three-dimensional (3D) Bose gas, we propose that the glass is a photonic bandgap material having a band with a dispersion relation resembling that of a massive particle.

Exploiting the phenomenon of thermal relaxation, two important applications are interesting to study: first, the conversion of a high-frequency pulse of light to a pulse of lower frequencies, distributed according to the Bose-Einstein statistics, and second, the generation of a multicolor light out of a monochromatic one, through an appropriate redistribution of the spectrum. We explore the possibility of using dielectric structures and multimode optical fiber gratings to realize the first application.

The paper is divided into four sections. The first is devoted to the physical description of the cavity and the second to the process of thermal relaxation leading to the frequency down-conversion. An extension to 3D cavity is studied, with a particular emphasis given to the case of fiber gratings, before ending with the conclusion.

II. DESCRIPTION OF THE CAVITY SYSTEM

We start with the description of the 2D setup. It consists of a planar microcavity surrounded by two semitransparent mirrors that ensure reflection of the longitudinal component of the electromagnetic field. The microcavity has a transverse size much larger than the longitudinal one, $L \sim \mu$m, and is filled with a material exhibiting significant third-order optical nonlinearity, together with a small absorption coefficient. Typically, let us take the case of a glass with linear and nonlinear refractive indices, $n_0 = 1.44$ and $n_2 \sim 3 \times 10^{-20}$ m^2/W, and a linear absorption coefficient $\alpha \sim 5 \times 10^{-5}$ m^{-1}. If the mirrors are perfect, longitudinal components $\vec{k}_\parallel$ of wave vector $\vec{k}$ take discrete values. The fundamental frequency corresponds to the lowest value $\omega_0 = 2\pi c/\lambda_0 = c/n_0|\vec{k}_{\parallel,0}| = c\pi/n_0 L$, where c is the velocity of light. Selecting only this lowest value and continuous values for transverse components $\vec{k}_\perp$, energy spectrum $\epsilon(\vec{k})$ of the photon moving in the cavity is that of a relativistic 2D Bose gas with a mass $m = \hbar\pi n_0/(Lc) \sim$ eV [3]:

$$\epsilon_{\vec{k}_\perp} = \hbar\omega_{\vec{k}_\perp} = \frac{\hbar c}{n_0}\sqrt{\left(\frac{\pi}{L}\right)^2 + \vec{k}_\perp^2} \simeq \hbar\omega_0 + \frac{\hbar^2 \vec{k}_\perp^2}{2m}. \qquad (1)$$

The perfection of the cavity is limited by the quality of the mirrors. High finesse of the order $\mathcal{F} \sim 10^6$ has been reported [4] for a dielectric having a thin layer structure made of SiO_2 and Ta_2O_5.

When an electric field propagates inside the microcavity, a polarization is created and is decomposed into three terms:

$$\vec{P}(\vec{x},t)=\vec{P}_{cons}(\vec{x},t)+\vec{P}_{dissip}(\vec{x},t)-\vec{P}_{gain}(\vec{x},t). \tag{2}$$

The first term conserves the electromagnetic energy:

$$\vec{P}_{cons}(\vec{x},t)=(\chi^{(1)}+\chi^{(3)}\vec{E}^2(\vec{x},t))\vec{E}(\vec{x},t) \tag{3}$$

and describes the linear and nonlinear polarizations that contribute to the total effective energy Hamiltonian:

$$H=\int_V d^3\vec{x}\left[\frac{\epsilon}{2}\vec{E}^2(\vec{x},t)+\frac{\mu_0}{2}\vec{H}^2(\vec{x},t)+\frac{3\chi^{(3)}}{4}\vec{E}^4(\vec{x},t)\right]. \tag{4}$$

The susceptibility coefficients are related to refractive index $n=n_0+n_2I$ through $\epsilon/\epsilon_0=n_0^2$ and $\chi^{(3)}=\epsilon_0^2(1+\chi^{(1)})cn_2$. The second term in Eq. (2) describes the dissipative losses $\vec{P}_{dissip}(\vec{x},t)=\alpha\vec{E}(\vec{x},t)$. Finally, the third term is present in the case where photons are created by an active medium inside the cavity, such as a semiconductor. In classical physics, the presence of these terms allows one to establish the following balance energy equation:

$$\frac{dH}{dt}=\int_V d^3\vec{x}[\vec{P}_{gain}(\vec{x},t)-\vec{P}_{dissip}(\vec{x},t)]\cdot\vec{E}(\vec{x},t). \tag{5}$$

The quantification of the hamiltonian amounts to replacing the coefficients coming from each mode of the fields by the corresponding creation and annihilation operators, $\hat{a}_i^\dagger$ and $\hat{a}_i$. Each index i labels a mode by its wave-vector components $\vec{k}$ and γ which represent the polarization state and distinguish between TE and TM waves [5]. Developing the Hamiltonian in terms of these operators, we make the rotating wave approximation by means of which we eliminate contribution that does not conserve the photon number. One obtains [3]

$$\hat{H}=\sum_i \hbar\omega_i\hat{a}_i^\dagger\hat{a}_i+\sum_{i,j,l,m}\frac{\mathcal{V}_{i,j,l,m}}{2V}\hat{a}_i^\dagger\hat{a}_j^\dagger\hat{a}_l\hat{a}_m. \tag{6}$$

The effective potential is assumed to be constant in a first approximation and is related to scattering length a through relation $\mathcal{V}_{i,j,l,m}=4\pi a\hbar^2/m$. This constant has been estimated in Ref. [3] and allows one to express the scattering length in terms of the nonlinear refractive index through:

$$a\simeq\frac{\hbar c^2n_2n_0^2}{4\pi}\left(\frac{2\pi}{\lambda_0}\right)^3. \tag{7}$$

For glass and $\lambda_0=1.5\,\mu$m, scattering length $a\sim3\times10^{-18}$ m is much lower than the value for atoms ($a_{atoms}\sim10^{-10}$ m) since the very low photon effective mass restricts the possibility of scattering.

III. FREQUENCY CONVERSION PROCESS

The frequency conversion inside the cavity proceeds in the following way. An initial photon pulse with a distribution centered in a frequency ω_p higher than ω_0 is created inside the cavity. This pulse could be generated from an active medium or could be injected from outside. Then, the thermalization process turns the initial distribution into a Bose-Einstein distribution $f(\omega_{\vec{k}_\perp})=[\exp(\beta\hbar\omega_{\vec{k}_\perp}-\beta\mu)-1]^{-1}$, presenting a narrow width maximum intensity at the lowest frequency ω_0, when the effective temperature $k_BT_{eff}=1/\beta$ is close to zero and chemical potential μ close to ω_0. No condensate transition is expected for a 2D free Bose gas, but quasicondensation is predicted for an interacting one. Thereafter, since the cavity finesse is finite, the resulting thermal distribution evolves slowly outward to obtain a final converted pulse outside the cavity.

The energy conservation imposes that part of the photons are distributed with a frequency higher than ω_p. If these photons escape from the cavity, the process of evaporative cooling increases the population in the lowest levels. This process takes place provided that a lower refractive index of the transverse coating allows the high transverse components of the light to escape from the transverse edge of the cavity. Assuming the minimum refractive index for the coating (air), only modes with a wavelength greater than $\lambda=\lambda_0/n_0$ (1 μm for $\lambda=1.5$ μm) remain confined inside the cavity. But, as seen below, evaporative cooling is not necessary to convert light frequency.

Full thermal relaxation of light requires that the photon gas remains confined and energetically isolated for a long enough time. Therefore, the average confining time of the photon inside the cavity must be much larger than the relaxation time towards equilibrium, τ_{relax}, which is the average time between two successive collisions of photons [6]. The confining time is limited by the absorption time inside the cavity, $\tau_{abs}=1/(\alpha c)$, and by cavity finesse $\mathcal{F}$ leading to a damping time of the field, $\tau_{cav}\sim L\mathcal{F}n_0/c$. For the case of a glass, the second effect is dominant since $\alpha^{-1}\gg\mathcal{F}L$ and therefore, the first realization condition states $\tau_{cav}\gg\tau_{relax}$.

The relaxation time can be estimated approximately from the kinetic theory to be

$$\frac{1}{\tau_{relax}}=\rho\frac{c}{n_0}\sigma(1+\overline{N}) \tag{8}$$

and depends directly on average photon density ρ, the velocity inside the medium, c/n_0, the total cross section σ (assumed to be constant as a function of the energy), and degeneracy factor $\overline{N}$ which is the average photon number per mode. Note that if $\hbar\to0$ then $a\to0$ and that no relaxation exists, since in a classical field system the photon does not exist.

The total cross section is calculated in the Born approximation for the constant potential, taking into account the conservation of momentum and the conservation of energy during the binary collision. The result is $\sigma=4\pi a^2$ [7].

Factor $\overline{N}$ is introduced to take into account the Bose enhancement that stimulates the collision process and diminishes the time to reach thermal equilibrium. By analogy with the laser, a high population in an excited mode state, N_{exc}, enhances the deexcitation process through stimulated emission. This effect combined with the spontaneous emission leads to a global enhancement factor $1+N_{exc}$. Similarly, the presence of a high degeneracy in population cavity mode $N_{\vec{k}_\perp}$ stimulates the quantum collision process through factor $1+N_{\vec{k}_\perp}$. This factor originates from the Uehling-Uhlenbeck quantum kinetic equation governing the evolution of modes population [6,8]. In comparison with the Boltzmann kinetic equation describing the relaxation of a classical gas, such factors appear in the binary collision rate in this quantum equation and amplify the relaxation process. As a consequence, the collision rate, which is linear in the mode population distribution in the case of the Boltzmann equation, becomes quadratic in the case of the Uehling-Uhlenbeck equation. That is why, in addition to the usual classical formula for relaxation, we introduce factor $1+\overline{N}$ in the estimation of Eq. (8). We estimate $\overline{N}$ as the ratio between average density ρ and the average density of mode assuming that the photon density is, on an average, uniformly distributed for any frequency between ω_0 and ω_p and is zero otherwise. In this approximation, the average density of mode is

$$\frac{1}{L}\int_{\mathcal{S}}\frac{d^2\vec{k}_\perp}{(2\pi)^2}=\frac{\pi}{L\lambda_0^2}\left[\left(\frac{\lambda_0}{\lambda_p}\right)^2-1\right], \qquad (9)$$

where $\mathcal{S}$ is the area of the wave vector satisfying $|\vec{k}_\perp|^2<(2\pi/\lambda_p)^2-(2\pi/\lambda_0)^2$. Using Eq. (9), we obtain

$$\overline{N}=\frac{L\lambda_0^2\rho}{\pi}\left[\left(\frac{\lambda_0}{\lambda_p}\right)^2-1\right]^{-1}. \qquad (10)$$

Important deviation from this uniformity occurs if the initial (final) distribution is narrowly peaked around the initial (final) frequency ω_p (ω_0).

The second realization condition is that the photon number or power absorbed inside the medium should not exceed a certain amount, in order not to increase, considerably, the temperature of the glass. If we authorize an increase of 1 K per pulse, and since the specific heat of glass is about 2 J/m^3 K, the photon density absorbed must not be greater than $\rho_l\sim 2\times 10^{25}$ photons/m^3. If $\tau_{cav}\ll\tau_{abs}$ then the fraction absorbed during time τ_{cav} is

$$\rho[1-\exp(-\tau_{cav}/\tau_{abs})]\simeq\rho\tau_{cav}/\tau_{abs}\leqslant\rho_l. \qquad (11)$$

Using the explicit expression for τ_{cav}, τ_{abs}, and Eqs. (8), (10), and (11), the two realization conditions combined together impose the following constraints on the density:

$$\left[\left(\frac{\lambda_0}{\lambda_p}\right)^2-1\right]^{1/2}\frac{1}{2L\lambda_0 a\sqrt{\mathcal{F}}}\ll\rho\leqslant\frac{\rho_l}{n_0\alpha L\mathcal{F}}. \qquad (12)$$

These inequalities express that the photon density must be high enough in order to favor collision rate over the damping rate, and must not exceed a critical value in order not to heat, considerably, the glass.

To satisfy these inequalities for a ratio $\lambda_0/\lambda_p=1.5$, the fidelity factor must obey $\mathcal{F}\leqslant 10^{14}$. In the realistic case of a dielectric with $\mathcal{F}\leqslant 10^6$, the minimum density corresponds to 10^{26} photons/m^3 or 2×10^{16} W/m^2. If the input light pulse time is $\tau_{cav}=2.5\times 10^{-9}$ s, we need a peak power of 10^6 W/cm^2.

The effective temperature and the chemical potential are estimated by expressing the conservation of the photon number and energy of the pulse before and after thermal relaxation, neglecting the much smaller photon and energy losses. The initial pulse is injected in such a way that the photon density ρ and energy density $\hbar\omega_p\rho$ fill, uniformly, the total volume of the cavity. The final pulse has a Bose-Einstein frequency distribution with the dispersion relation (1). We deduce the following balance equations:

$$\rho=\frac{1}{L}\int_{\mathcal{S}}\frac{d^2\vec{k}_\perp}{(2\pi)^2}f(\omega_{\vec{k}_\perp})=\frac{2}{\lambda_B^2 L}\left(g_1(\kappa)+\frac{g_2(\kappa)}{\beta\hbar\omega}\right), \qquad (13)$$

$$\hbar\omega_p\rho=\frac{2}{L}\int_{\mathcal{S}}\frac{d^2\vec{k}_\perp}{(2\pi)^2}\hbar\omega_{\vec{k}_\perp}f(\omega_{\vec{k}_\perp})$$
$$=\hbar\omega_0\rho+\frac{2}{\beta\lambda_B^2 L}\left(g_2(\kappa)+2\frac{g_3(\kappa)}{\beta\hbar\omega}\right), \qquad (14)$$

where $\lambda_B=h/\sqrt{2\pi m k_B T}$ is the thermal wavelength and $\kappa=\beta(\hbar\omega_0-\mu)$. $g_k(z)=\Sigma_{j=1}^{\infty}e^{-jz}/j^k$ are the Bose-Einstein functions. The factor of 2 takes into account the two states of polarization. The resulting thermal pulse has an effective temperature of $T_{eff}\sim 10^6$ K and a chemical potential $\mu\to\hbar\omega_0$. In this situation, most of the light is redistributed in the low-frequency region. If evaporative cooling takes place, the final temperature is lowered since some high-energy photons are lost and, consequently, the conversion rate decreases.

In the case of multimode fiber gratings, an all-in-one integrated structure is composed of two mirrors and the microcavity (Fig. 1). Three typical values of parameters are $\mathcal{F}=10^3$, $\alpha=5\times 10^{-2}$ m^{-1}, and the transverse core section of 10^2 μm^2, allowing one to populate higher frequency transverse modes if the cladding index is close to unity [9,10]. This corresponds to a minimum density of 10^{18} W/m^2 and $\tau_{cav}=2.5\times 10^{-12}$ s and a peak power of 10^5 W. The use of such a power has been reported in fiber grating experiments [11].

IV. EXTENSION TO A 3D CAVITY

In order to create a real condensate, a 3D cavity is needed, which must allow continuous values of the longitudinal wave-vector component. One possibility is that the cavity is itself a photonic band-gap material made of periodic layers, in such a way that ω_0 is the minimum frequency of an allowed band with wave vector $\vec{k}_{\parallel,0}$. In general, the interband interaction between particles is less frequent than the intraband interaction which, due to the difficulty in satisfying momentum-energy conservation, has much less probability to occur. Therefore, the thermal relaxation process is mainly achieved inside the band and generates a real condensate. We assume that between ω_0 and ω_p, the frequency spectrum

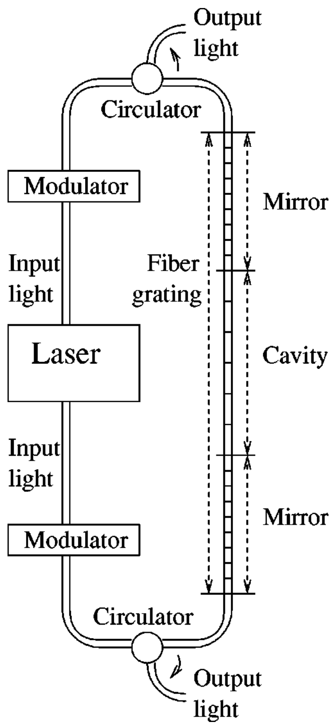

FIG. 1. Setup for frequency down-conversion using fiber grating.

within the band is of the form

$$\omega(\vec{k})=\omega_0+\frac{\hbar(\vec{k}-\vec{k}_{\parallel,0})^2}{2m}. \tag{15}$$

As a consequence, the integration over the whole wave-vector space, with frequency between ω_0 and ω_p, changes the average density of mode in 3D into $4\pi[2\omega_0(\omega_p-\omega_0)]^{3/2}$. For a longitudinal size $\tilde{L}$ of the photonic band-gap material, the damping time is modified into $\tau_{cav}\sim\tilde{L}\mathcal{F}n_0/c$ and the constraints on the density become, in 3D,

$$\left[\left(\frac{\lambda_0}{\lambda_p}\right)-1\right]^{3/4}\frac{1}{\lambda_0^{3/2}a\sqrt{\mathcal{F}\tilde{L}}}\ll\rho\leqslant\frac{\rho_l}{n_0\alpha\tilde{L}\mathcal{F}}. \tag{16}$$

For a photonic band gap integrated inside the fiber grating with $\tilde{L}=10^3\ L$ and $\mathcal{F}=10^3$, we obtain a lower photon density of 2×10^{16} W/m^2, a much higher $\tau_{cav}=2.5\times10^{-9}$ s, and a lower peak power of 10^3 W, compared to the case of 2D. Using the dispersion relation (15), the effective temperature and the condensate population ρ_0 are estimated from the correponding photon number and energy balance equations in 3D. Integrating over the whole space of wave vector, we extend Eqs. (13) and (14) in 3D and deduce

$$\rho=\rho_0+2\zeta\left(\frac{3}{2}\right)\frac{1}{\lambda_B^3}, \tag{17}$$

$$\hbar(\omega_p-\omega_0)\rho=2\zeta\left(\frac{5}{2}\right)\frac{k_BT_{eff}}{\lambda_B^3}, \tag{18}$$

where ρ_0 is the condensate density and $\zeta(x)$ the Riemann zeta function. We find $T_{eff}\sim10^6$ K much lower than critical temperature $T_c\sim10^8$ K, which means that $\rho_0\simeq\rho$. The uncertainty in frequency ω_0 in the resulting condensate is limited by the cavity finesse through $\Delta\omega\sim1/\tau_{cav}$.

Strictly speaking, these estimations need to be validated by a more detailed description of the dynamical process. But at present, we prefer a model based on a simple estimation for two reasons. First, adequate kinetic equations must be formulated which take into account the presence of the condensate. Various models for atomic particle exist in the literature but no one is commonly accepted [8]. In this context, there is no advantage to use one more sophisticated model if it is not justified by experiment. Second, the particular nonlinear properties of a fiber grating at high light intensity are not yet well known to allow a complete characterization [11,12].

Other important nonlinear effects are the Raman and the Brillouin scatterings which can compete to convert the frequencies and can increase the absorption loss inside the cavity [9]. The rate at which these phenomena occur in an ordinary glass are estimated to be linearly proportional to the density, with constants 10^{-13} m/W and 6×10^{-11} m/W for the peak Raman gain, and peak Brillouin gain respectively [9]. For the power of light considered, this corresponds to relevant rates of 10^{12}–10^{14} Hz which contribute to broadening of the spectrum of the input pulse centered around ω_p.

Another important question is how to inject an initial pulse of light inside the cavity. If we exclude the use of an active medium, the light can be introduced transversely on the transverse edge, by coupling the cavity medium with an optical fiber or with a prism [13]. In the case of fiber gratings, Fig. 1 depicts a possible setup. Two initial pulses, generated by a laser, are modulated to get a specific spatial profile in order to enter coherently in the fiber grating through the mirrors. After the frequency conversion takes place, the resulting light distribution leaves the cavity and gets out by means of the circulators.

This spatial profile of the pulses must be designed in order to inject a maximum percentage of the intensity inside the cavity. At least in the case of small light intensity it is possible to determine an optimal profile through a dynamical simulation. Indeed, we notice that the time-reversal process describes an initial light density inside the cavity evolving towards two pulses outside the cavity. This process is governed by the linear classical Maxwell equation and thus, can be simulated. Since the dynamic evolution is time-reversal invariant when the dissipation is neglected, the spatial profile of the two resulting pulses is precisely the one to be used in order to confine the totality of the light inside the cavity. The size of the profile for each pulse is determined by the damping rate and is of the order of $\tau_{cav}c/n_0$.

V. CONCLUSION

In conclusion, we show the feasibility to convert the frequency of a light inside an optical nonlinear cavity. The analysis is based on an estimation of the collision rate between photons in a glass, leading to a thermal equilibrium and a high condensed population in the fundamental mode of the cavity. Fiber gratings are possible devices to be used for the conversion. But a better experimental knowledge of nonlinear properties of fiber gratings is needed and an improved modeling should be developed for a more precise description of the various dynamical processes involved.

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