

Proposal for a Loophole-Free Bell Test Using Homodyne Detection

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We propose a feasible optical setup allowing for a loophole-free Bell test with efficient homodyne detection. A non-Gaussian entangled state is generated from a two-mode squeezed vacuum by subtracting a single photon from each mode, using beam splitters and standard low-efficiency single-photon detectors. A Bell violation exceeding 1% is achievable with 6 dB squeezed light and a homodyne efficiency around 95%. A detailed feasibility analysis, based upon the recent experimental generation of single-mode non-Gaussian states, suggests that this method opens a promising avenue towards a complete experimental Bell test.

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In their seminal 1935 paper, Einstein, Podolsky, and Rosen (EPR) advocated that if “local realism” is taken for granted, then quantum theory is an incomplete description of the physical world [1]. The EPR argument gained a renewed attention in 1964 when Bell derived his famous inequalities, which must be satisfied within any local-realistic theory [2]. The violation of Bell inequalities (BI), predicted by quantum mechanics, has since then been observed in many experiments [3–7], disproving the concept of local realism. So far, however, all these tests suffered from “loopholes” allowing a local-realistic explanation of the experimental observations by exploiting either the low detector efficiency [8,9] or the timelike interval between the detection events [10,11]. Several schemes were proposed which circumvent these loopholes based on entangled pairs of photons [12,13], Hg atoms [14], Rydberg atoms [15], or trapped ions [16], but all presented a formidable experimental challenge. In this Letter, we propose a scheme based on the conditional generation of non-Gaussian entangled light states and balanced homodyning, which is experimentally realistic and allows for a “loophole-free” Bell test.

A test of the violation of BI typically involves two distant parties, Alice and Bob, who simultaneously carry out measurements on two systems prepared in an entangled quantum state. The measurement events (including the measurement choice) at Alice’s and Bob’s sites must be spacelike separated in order to avoid the locality loophole [10,11], that is, to rule out any possible communication between the two parties. Optical systems are therefore particularly suitable candidates for Bell tests because entangled photon pairs can now be generated and distributed over long distances [5,6]. However, the currently available single-photon detectors suffer from too low efficiencies η , opening the so-called detector-efficiency loophole [8,9]; that is, the experimental data can be explained by local-realistic theories wherein the detectors click only with probability η . This loophole is present in all optical Bell tests today.

In contrast, very high detection efficiencies can be reached in optical systems using balanced homodyne detectors [17–19], opening a very promising alternative to Bell tests based on single-photon detectors. Several theoretical works have shown that a violation of BI may indeed be observed with balanced homodyning provided that some very specific entangled light states can be prepared [20–22]. Although a violation up to the maximum theoretical limit can be achieved with homodyne detection [22], all the states required in [20–22] unfortunately appear to be experimentally infeasible. On the other hand, the experimentally accessible [23–25] two-mode squeezed vacuum states are unsuitable for a Bell test because they are characterized by a positive-definite Gaussian Wigner function, which provides an explicit hidden-variable model for homodyne measurements. So far, no feasible experimental scheme has been found that could be used to prepare non-Gaussian states that exhibit a violation of BI with balanced homodyning.

In this Letter, we show that a strikingly simple optical setup (see Fig. 1) can be used to conditionally generate non-Gaussian states that are suitable for this purpose. Our scheme requires a pulsed source of two-mode squeezed vacuum state, which can be expressed in the Fock basis as $|\psi_{\text{in}}\rangle_{AB} = \sqrt{1 - \lambda^2} \sum_{n=0}^{\infty} \lambda^n |n, n\rangle_{AB}$, where $\lambda = \tanh(r)$ and r is the squeezing constant. This state is “degaussified” by subtracting a photon from each mode [26–28]. More precisely, we produce a non-Gaussian entangled state with the use of two unbalanced beam splitters BS_A and BS_B with intensity transmittance T , followed by two single-photon detectors PD_A and PD_B , such as avalanche photodiodes. The successful state preparation is heralded by a click of both PD_A and PD_B . We shall see that a detector efficiency as low as $\eta = 10\%$ suffices for the preparation of states exhibiting a violation of BI.

The single-photon subtraction requires a high transmittance of BS_A and BS_B , since then the most probable event leading to a click at each PD is when a single-photon is reflected by each BS. In the limit $T \rightarrow 1$, the

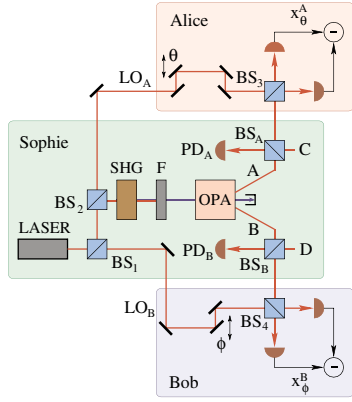


FIG. 1 (color online). Proposed experimental setup. The source (controlled by Sophie) is based on a master laser beam, which is converted into second harmonic in a nonlinear crystal (SHG). After spectral filtering (F), the second harmonic beam pumps an OPA which generates two-mode squeezed vacuum in modes A and B . Single photons are conditionally subtracted from modes A and B with the use of the beam splitters BS_A and BS_B and single-photon detectors PD_A and PD_B . Alice (Bob) measures a quadrature of mode A (B) using a balanced homodyne detector that consists of a balanced beam splitter BS_3 (BS_4) and a pair of highly efficient photodiodes. The local oscillators LO_A and LO_B are extracted from the laser beam by means of two additional beam splitters BS_1 and BS_2 . The random switching of the relative phase θ (ϕ) between LO_A and A (LO_B and B) can be performed using fast electro-optical modulators.

conditionally generated state is very close to the pure state $|\psi_{\text{out}}\rangle_{AB} \propto \hat{a}_A \hat{a}_B |\psi_{\text{in}}\rangle_{AB} \propto \sum_{n=0}^{\infty} (n+1)(T\lambda)^n |n, n\rangle_{AB}$, where $\hat{a}_{A,B}$ are annihilation operators. This state is non-Gaussian, and therefore suitable for a Bell test based on homodyning. Note that the generation of pulsed single-mode non-Gaussian states by photon subtraction from squeezed states has recently been demonstrated experimentally by some of us [29], so the preparation of two-mode non-Gaussian states should be feasible along the same lines.

In the experiment proposed here, Alice and Bob measure the quadratures $x_{\theta}^A = \cos\theta x^A + \sin\theta p^A$ and $x_{\phi}^B = \cos\phi x^B + \sin\phi p^B$, which have continuous spectra and satisfy $[x^j, p^k] = i\delta_{jk}$. In order to use the Clauser-Horne-Shimony-Holt (CHSH) inequality [30], we discretize x_{θ}^A and x_{ϕ}^B by postulating that the outcome is $+1$ when $x \geq 0$, and -1 otherwise. Alice and Bob must then choose randomly and independently between two different measurements, corresponding to the choices of angles $\theta = \theta_1$ or θ_2 and $\phi = \phi_1$ or ϕ_2 . Their measurement outcomes are thus described by four variables a_1, a_2, b_1, b_2 , with values $+1$ or -1 . One defines the usual Bell parameter

$$S = \langle a_1 b_1 \rangle + \langle a_1 b_2 \rangle + \langle a_2 b_1 \rangle - \langle a_2 b_2 \rangle, \quad (1)$$

where $\langle a_j b_k \rangle$ denotes the average over the subset of the experimental data when Alice measured a_j and, simulta-

neously, Bob measured b_k . As we shall see, exploiting the fact that PD_A and PD_B can be viewed here as “event-ready” detectors [10], one can prove that all local-realistic models for Alice and Bob measurements must satisfy the Bell-CHSH inequality $|S| \leq 2$.

In the formalism of event-ready detectors introduced by Bell [10], one should know, by some initiating event, when a measurable system has been produced. The main idea is to preselect—rather than post-select—the relevant events. For that purpose, one considers three partners: Alice and Bob, who perform the measurements, and Sophie, who controls the source (see Fig. 1). The entire data analysis must be performed on a pulsed basis, with Sophie sending time-tagged light pulses (local oscillator and squeezed light) to Alice and Bob. In each experimental run, Sophie records whether her photodetectors clicked, while Alice and Bob carry out spacelike separated measurements of one of two randomly chosen quadratures. After registering a large number of events, the three partners discard all events not corresponding to an event-ready double click registered by Sophie. The correlation coefficients $\langle a_j b_k \rangle$ are then evaluated from all remaining events and plugged into the S parameter (1).

In a local-realistic approach, the light pulses in each time slot supposedly carry some random unknown parameter μ , which ultimately determines the sign of x_{θ}^A and x_{ϕ}^B . Imposing, by proper timing, that the clicks of Sophie’s conditioning detectors cannot be influenced by the measurements on Alice’s and Bob’s sides implies that the probability distribution $p(\mu)$ is independent of the measurement phases $\theta_{1,2}$ and $\phi_{1,2}$. The measured sign s_A on Alice’s side (s_B on Bob’s side) therefore depends only on μ and θ (ϕ on Bob’s side), so that $\langle a_j b_k \rangle = \int d\mu p(\mu) s_A(\theta_j, \mu) s_B(\phi_k, \mu)$, from which the derivation of the Bell-CHSH inequality is very standard [31]. Consequently, a truly loophole-free Bell test can be performed provided that Sophie’s event-ready detectors effectively preselect the measuring events.

Let us now evaluate the quantum mechanical value of the S parameter in the proposed experimental scheme. We only briefly outline the main steps of the calculation, as the details will be presented elsewhere [32]. We model realistic photodetectors (which have a limited quantum efficiency $\eta < 1$, and cannot discriminate between one and multiple photon detection events) as ideal detectors preceded with a “virtual” beam splitter of transmittance η . Ideal detectors respond with two different outcomes, either a no click (projection onto the vacuum) or a click (projection on all states with at least one photon). Similarly, a balanced homodyne detector with efficiency η_{BHD} is modeled as a perfect homodyne detector preceded with a virtual beam splitter of transmittance η_{BHD} . At the output of the optical parametric amplifier, the modes A and B are prepared in a two-mode squeezed vacuum state, and the auxiliary modes C and D are in

vacuum state. The Wigner function of modes A, B, C, D is a Gaussian function centered at the origin,

$$W_{\text{in},ABCD}(r) = \frac{1}{\pi^4 \sqrt{\det \gamma_{\text{in}}}} \exp[-r^T \gamma_{\text{in}}^{-1} r]. \quad (2)$$

This state is fully characterized by the covariance matrix $\gamma_{ij} = \langle r_i r_j + r_j r_i \rangle - 2\langle r_i \rangle \langle r_j \rangle$, where $r = (x^A, p^A, \dots, x^D, p^D)$ is a vector of quadrature components. The input covariance matrix is $\gamma_{\text{in}} = \gamma_{\text{in},AB} \oplus I_{CD}$, where I is the identity matrix and $\gamma_{\text{in},AB}$ denotes the covariance matrix of the two-mode squeezed vacuum $|\psi_{\text{in}}\rangle_{AB}$. After combining modes A and C (B and D) on an unbalanced beam splitter BS_A (BS_B) with transmittance T , the mode C (D) is detected by the photon-counting detector PD_A (PD_B), while mode A (B) is sent to the homodyne detector. The covariance matrix γ_{out} of the mixed Gaussian state of modes A, B, C, D after passing through the beam splitters BS_A and BS_B and the four virtual beam splitters modeling the imperfect detectors is related to γ_{in} via a Gaussian completely positive map $\gamma_{\text{out}} = S_\eta S_{\text{BS}} \gamma_{\text{in}} S_{\text{BS}}^T S_\eta^T + G$, where S_{BS} describes the mode coupling in BS_A and BS_B , and the matrices $S_\eta = \sqrt{\eta_{\text{BHD}}} I_{AB} \oplus \sqrt{\eta} I_{CD}$ and $G = (1 - \eta_{\text{BHD}}) I_{AB} \oplus (1 - \eta) I_{CD}$ account for the imperfect detectors [32].

The Wigner function of the state of modes A and B prepared by conditioning on observing clicks at PD_A and PD_B can then be expressed as a linear combination of four Gaussian functions,

$$W(r) = \frac{1}{\pi^2 P \sqrt{\det \gamma_{\text{out}}}} \sum_{j=1}^4 \frac{q_j}{\sqrt{\det \Gamma_{j,CD}}} \exp[-r^T \Gamma_j r], \quad (3)$$

where $P = (\det \gamma_{\text{out}})^{-1/2} \sum_{j=1}^4 q_j [\det(\Gamma_j \Gamma_{j,CD})]^{-1/2}$ stands for the probability of success, $r = (x^A, p^A, x^B, p^B)$, and we have defined $q_1 = 1, q_2 = q_3 = -2, q_4 = 4$. The various matrices appearing in Eq. (3) are obtained from γ_{out}^{-1} ,

which can be split into four smaller submatrices as $\gamma_{\text{out}}^{-1} = \begin{bmatrix} \Gamma_{AB} & \sigma \\ \sigma^T & \Gamma_{CD} \end{bmatrix}$. These four submatrices are then used to define $\Gamma_j = \Gamma_{AB} - \sigma \Gamma_{j,CD}^{-1} \sigma^T$, where $\Gamma_{1,CD} = \Gamma_{CD}, \Gamma_{2,CD} = \Gamma_{CD} + I_C \oplus 0_D, \Gamma_{3,CD} = \Gamma_{CD} + 0_C \oplus I_D,$ and $\Gamma_{4,CD} = \Gamma_{CD} + I_{CD}$.

After discretization of the quadratures, the correlation coefficient can be expressed as

$$\langle a_j b_k \rangle = \int_{-\infty}^{\infty} \text{sign}(x_{\theta_j}^A x_{\phi_k}^B) P(x_{\theta_j}^A, x_{\phi_k}^B) dx_{\theta_j}^A dx_{\phi_k}^B, \quad (4)$$

where $P(x_{\theta_j}^A, x_{\phi_k}^B)$ is the joint probability distribution of the two commuting quadratures $x_{\theta_j}^A$ and $x_{\phi_k}^B$, which can be determined as a marginal distribution from the Wigner function given by Eq. (3). The correlation coefficient (4)—and therefore S —can then be expressed in a closed form by analytically integrating the resulting Gaussian functions [32].

The results plotted in Fig. 2 were obtained for the optimal choice of angles $\theta_1 = 0, \theta_2 = \pi/2, \phi_1 = -\pi/4, \phi_2 = \pi/4$. Figure 2(a) shows that the Wigner function W is negative in some regions of the phase space, which is a necessary condition for the observation of a violation of BI with homodyne detectors. Figure 2(b) illustrates that the Bell inequality $|S| \leq 2$ can indeed be violated with the proposed setup, and shows that there is an optimal squeezing λ_{opt} which maximizes S . A simplified calculation assuming perfect detectors with single-photon resolution [32] predicts that $\lambda_{\text{opt}} T \approx 0.57$. It follows from Fig. 2(c) that the Bell factor S depends only very weakly on the efficiency η of the single-photon detectors, so the Bell inequality can be violated even if $\eta \lesssim 10\%$. In contrast, the efficiency of the homodyne detectors η_{BHD} must be above $\sim 90\%$; see Fig. 2(d).

The maximum Bell parameter achievable with our scheme is about $S_{\text{max}} \approx 2.046$, which represents a BI violation of 2.3%. This is a small, but statistically significant violation, which should be possible to observe experimentally. The required degree of squeezing to get close to S_{max} corresponds to $\lambda \approx 0.57$, i.e., approximately 5.6 dB, a value that has already been achieved experimentally [17,33]. Another important parameter is the transmittance T of the beam splitters BS_A and BS_B , which must be as high as possible for maximizing S . However, the probability of successful state preparation can be estimated as $P \approx \eta^2(1-T)^2$, so that it quickly drops

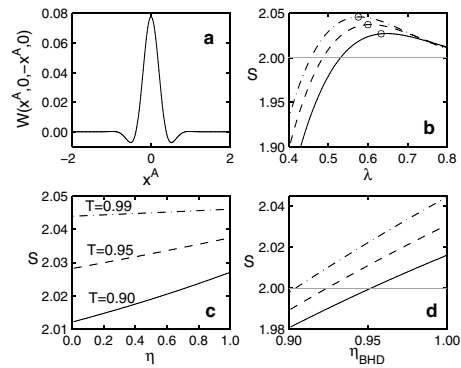


FIG. 2. Violation of Bell-CHSH inequality with the conditionally prepared non-Gaussian state. (a) One-dimensional cut of the Wigner function of the conditionally generated non-Gaussian two-mode state (with $\lambda = 0.5, T = 0.95$, and $\eta = 30\%$) along the line $x^B = -x^A, p^A = p^B = 0$. Notice the regions where W is negative. (b) Bell parameter S as a function of the squeezing λ of the initial two-mode squeezed vacuum. The curves are plotted for perfect detectors ($\eta = \eta_{\text{BHD}} = 100\%$) with $T = 0.9$ (solid line), $T = 0.95$ (dashed line), and $T = 0.99$ (dot-dashed line). The open circles mark the points where $T\lambda = 0.57$. (c) Bell parameter S as a function of the efficiency η of the single-photon detectors, for $\lambda T = 0.57$ and $\eta_{\text{BHD}} = 100\%$. (d) Bell parameter S as a function of the efficiency η_{BHD} of the balanced homodyne detectors, for $\lambda T = 0.57$ and $\eta = 30\%$.

when T approaches unity. Thus, there is a trade off between S and P , which needs to be optimized by taking into account the statistical uncertainties of the data.

In order to be more specific, let us consider the single-mode photon subtraction experiment [29]. It is based on a commercial cavity-dumped titanium-sapphire laser, delivering nearly Fourier-limited pulses at 850 nm, with a duration of 150 fs and a repetition rate of 790 kHz. Squeezed vacuum pulses generated by parametric deamplification are sent through a beam splitter, and the reflected beam is detected by a silicon avalanche photodiode (APD). Conditional on a click, the transmitted pulse is prepared in a non-Gaussian state, which is measured by homodyne detection with an overall efficiency $\eta_{\text{BHD}} \approx 75\%$. This experiment gives us useful estimates for a possible Bell test. First, the delay between pulses (1.2 μs) allows ample time for individual pulse analysis. A fast random choice of the analyzed quadratures can be performed using electro-optical modulators on the LO beams, triggered for instance by digitizing the shot noise of locally generated auxiliary beams. Switching times around 100 ns, associated with propagation distances of a few tens of meters, seem quite feasible. The APDs can be triggered only when a pulse is expected, reducing the effect of dark counts to a negligible value. The intrinsic APD efficiency is about 50%, but the filtering used to select a single-mode currently reduces the overall η to less than 5%, which should be improved for accumulating enough statistics.

In conclusion, let us define a set of realistic parameter values that should be reached in a loophole-free Bell test: with $\eta = 30\%$, $T = 95\%$, and $\lambda = 0.6$, a violation of BI by about 1% should be observable if the homodyne efficiency η_{BHD} is larger than 95% [see Fig. 2(d)]. With a repetition rate of 1 MHz and $P \approx 2.6 \times 10^{-4}$, the number of data samples would be several hundreds per second, so that the required statistics to see a violation in the percent range could be obtained in a reasonable time (a few hours). In addition, the electronic noise of the homodyne detectors should be 15–20 dB below shot noise, which is attainable with low-noise charge amplifiers. All these numbers have already been reached separately in various experiments, but attaining them simultaneously certainly represents a serious challenge. Nevertheless, taking into account many possible experimental improvements, the existence of an experimental window for a loophole-free test of Bell inequalities can be considered as highly plausible. Therefore, it appears that, with quantum continuous variables, a reasonable compromise can be found between the experimental constraints and the very stringent requirements imposed by a loophole-free test of Bell inequalities.

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Note added.—After this work was completed, we have learned that a similar scheme has just been independently proposed by H. Nha and H. J. Carmichael [34].

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