Proposal for loophole-free Bell test using homodyne detection

R. García-Patrón*, J. Fiurášek*† and N.J. Cerf*

*QUIC, Ecole Polytechnique, CP 165, Université Libre de Bruxelles, 1050 Bruxelles, Belgium
†Department of Optics, Palacký University, 17. listopadu 50, 77200 Olomouc, Czech Republic

Abstract. We propose a feasible optical setup allowing for a loophole-free Bell test with efficient homodyne detection. A non-Gaussian entangled state is generated from a two-mode squeezed vacuum by subtracting a single photon from each mode, using beam splitters and standard low-efficiency single-photon detectors. A Bell violation exceeding 1% is achievable with 6 dB of squeezing, a photodetector efficiency around 10%, and an homodyne efficiency around 95%. A feasibility analysis, based upon the recent generation of single-mode non-Gaussian states via photon subtraction, suggests that this method opens a promising avenue towards a complete experimental Bell test.

INTRODUCTION

The discrepancy between quantum mechanics and the concept of "local realism" has puzzled physicists since the seminal paper by Einstein, Podolsky, and Rosen (EPR) [1]. The discussion concerning local realism gained a firm ground when John Bell derived inequalities which must be satisfied within the framework of any local realistic theory while quantum mechanics predicts their violation under certain circumstances [2]. Since then, the violation of Bell inequalities has been observed in many experiments. However, all these experimental tests suffered from either a low efficiency of the detectors [3] or the measurements were not space-like separated as required in a proper Bell test [4]. Due to the resulting so-called detector-efficiency loophole and locality loophole, the observed experimental data may be in fact explained in terms of local realistic theories.

Two most promising alternatives of performing a loophole-free Bell test consist of either closing the detection loophole in an optical experiment or closing the locality loophole in an experiment with trapped ions. Recently, it was proposed theoretically to entangle two distant trapped ions via entanglement swapping by first preparing an entangled state of an ion and a photon on each side and then projecting the two photons on a singlet state [5]. This way, the locality loophole in the Bell test with ions could be closed, but the entanglement swapping would require interference of two photons emitted by two different ions, which is experimentally very challenging.

In the optical experiments, it is quite hard to achieve the very high efficiency of single-photon detectors required for a loophole-free Bell test. On the other hand, a balanced homodyne detector can exhibit a very high detection efficiency [6], which could close the detector loophole. However, in order to observe Bell violation with homodyning, one has to use a state having a Wigner function which is negative in some regions of the phase space, because otherwise the Wigner function would provide an explicit local hidden variable model. In particular, this implies that the entangled two-mode squeezed vacuum state with positive Gaussian Wigner function cannot be directly employed in a Bell test with homodyne detectors. It was shown theoretically that Bell inequality violation with homodyning can be observed if very specific entangled light states such as pair-coherent states [7] or specifically tailored finite superpositions of Fock states [8, 9], are available. However, no feasible experimental scheme is known that could generate those states.

Recently, we have shown that a surprisingly simple and experimentally feasible non-Gaussian state obtained from two-mode squeezed vacuum by conditionally removing a single photon from each mode can exhibit Bell violation with homodyning [10]. An essential feature of our proposal is that the photon subtraction can be successfully performed with low-efficiency single-photon detectors, such as the commonly used avalanche photodiodes. In fact, the basic element of our setup, the subtraction of a single photon from a single-mode squeezed vacuum, has been recently successfully implemented experimentally [11]. In what follows, we describe the proposed setup and briefly present the main results, the details can be found in [10, 12].

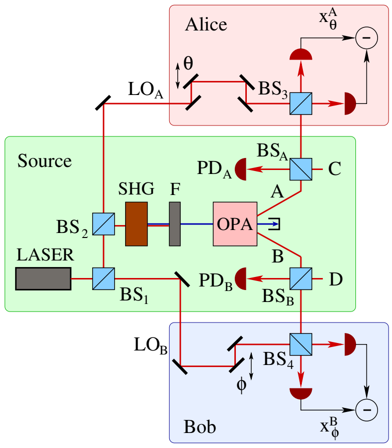

FIGURE 1. Proposed experimental setup.

BELL-CHSH INEQUALITY

The usual experimental setup for a test of Bell inequality violation involves two distant parties, Alice and Bob, who simultaneously perform measurements on parts of a shared bipartite quantum system that is distributed to them from some source. Alice and Bob should randomly and independently choose between one of two different quantum measurements a_1, a_2 and b_1, b_2. In the simplest scenario, each of these measurements has only two possible outcomes, $+1$ or -1. In this case, the incompatibility of the experimental data with local realism can be demonstrated with the help of the Bell-CHSH inequality [13]. Let us introduce the Bell factor S which is a linear combination of the correlations between Alice's and Bob's data,

$$S = \langle a_1 b_1 \rangle + \langle a_1 b_2 \rangle + \langle a_2 b_1 \rangle - \langle a_2 b_2 \rangle. \tag{1}$$

Here $\langle a_j b_k \rangle$ denotes the average over the subset of experimental data where Alice measured a_j and, simultaneously, Bob measured b_k. If the observed correlations can be explained within the framework of the local-hidden variable theories, then S must satisfy the Bell-CHSH inequality $|S| \leq 2$.

PROPOSED EXPERIMENTAL SETUP

The proposed experimental setup is shown in Figure 1. The two-mode squeezed vacuum in modes A and B is generated in a non-degenerate optical parametric amplifier (OPA) which is pumped by a second-harmonic of the master laser frequency doubled in a nonlinear crystal (SHG) and spectrally filtered (F). A single photon is subtracted from each mode A and B [14, 15, 16] with the use of an unbalanced beam splitter BS$_A$ (BS$_B$) with a high transmittance T∼0.90 which reflects a small part of the beam onto a photodetector PD$_A$ (PD$_B$). A successful photon subtraction is heralded by a click of both photodetectors PD$_A$ and PD$_B$. Note that commonly employed photodetectors exhibit a single-photon sensitivity but not a single photon resolution. This, however, is not a problem here because in the limit of high T the most probable event leading to the click of a photodetector is precisely that a single photon has been reflected from the squeezed beam on the beam splitter.

After generation of the non-Gaussian state, the beam A (B) together with the appropriate local oscillator LO$_A$ (LO$_B$) is sent to Alice (Bob), who then randomly and independently measures one of two rotated quadratures $x^A_{\theta_{1,2}} = x^A \cos\theta_{1,2} + p^A \sin\theta_{1,2}$ ($x^B_{\phi_{1,2}} = x^B \cos\phi_{1,2} + p^B \sin\phi_{1,2}$) where the relative phase $\theta_{1,2}$ ($\phi_{1,2}$) between the beam A (B) and the local oscillator LO$_A$ (LO$_B$) can be switched, e.g., by means of a fast electro-optical modulator. In order to use the Bell-CHSH inequality, the quadratures are converted into binary outcomes as follows, $a_{1,2} = \text{sign}(x^A_{\theta_{1,2}})$ and $b_{1,2} = \text{sign}(x^B_{\phi_{1,2}})$, where $\text{sign}(x) = 1$ if $x > 0$ and $\text{sign}(x) = -1$ if $x \leq 0$. To avoid the locality loophole, the whole experiment has to be carried out in a pulsed regime and a proper timing is necessary. In particular, the measurement events on Alice's and Bob's sides (including the choice of phases) have to be space-like separated, and cannot influence the "event-ready" detectors PD$_A$ and PD$_B$.

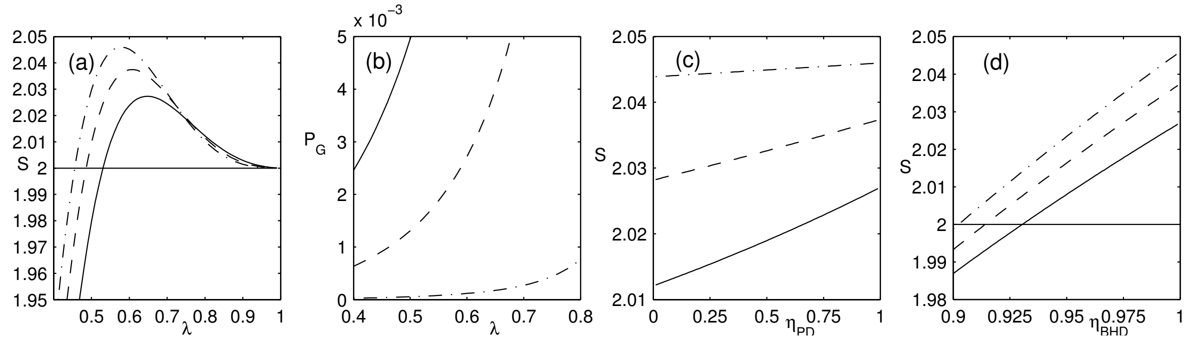

FIGURE 2. Violation of Bell-CHSH inequality with the conditionally-prepared non-Gaussian state. (a) Bell parameter S as a function of the squeezing λ of the initial two-mode squeezed vacuum. The curves are plotted for perfect detectors ($\eta = \eta_{\mathrm{BHD}} = 100\%$) with $T = 0.9$ (solid line), $T = 0.95$ (dashed line), and $T = 0.99$ (dot-dashed line). (b) Probability P_G of successful generation of the non-Gaussian state for perfect detectors. Considering realistic single-photon detectors, the probability would be reduced by a factor η_{PD}^2. (c) Bell parameter S as a function of the efficiency η_{PD} of the single-photon detectors, for $\lambda T = 0.57$ and $\eta_{\mathrm{BHD}} = 100\%$. (d) Bell parameter S as a function of the efficiency η_{BHD} of the balanced homodyne detectors, for $\lambda T = 0.57$ and $\eta_{\mathrm{PD}} = 30\%$.

After registering a large number of events, the partners discard all events not corresponding to an "event-ready" double-click of PD_A and PD_B. From the remaining events they calculate the four correlations $\langle a_j b_k \rangle$ and check if the Bell factor S violates the Bell-CHSH inequality.

RESULTS

The correlation $\langle a_j b_k \rangle$ can be calculated from the joint probability distribution of the quadratures $x^A_{\theta_j}$ and $x^B_{\phi_k}$, $P(x^A_{\theta_j}, x^B_{\phi_k}) \equiv \langle x^A_{\theta_j}, x^B_{\phi_k} | \rho_{c,AB} | x^A_{\theta_j}, x^B_{\phi_k} \rangle$, where $\rho_{c,AB}$ denotes the (normalized) conditionally generated non-Gaussian state of modes A and B. We have

$$\langle a_j b_k \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{sign}(x^A_{\theta_j} x^B_{\phi_k}) P(x^A_{\theta_j}, x^B_{\phi_k}) dx^A_{\theta_j} dx^B_{\phi_k}, \tag{2}$$

The joint probability $P(x^A_{\theta_j}, x^B_{\phi_k})$ can be obtained as a marginal distribution from the Wigner function of $\rho_{c,AB}$, which can be expressed as a linear combination of four Gaussian functions. For a detailed calculation, see [12].

The results plotted in Figure 2 were obtained for the optimal choice of angles $\theta_1 = 0$, $\theta_2 = \pi/2$, $\phi_1 = -\pi/4$, $\phi_2 = \pi/4$. Figure 2 (a) illustrates the dependence of the Bell factor S on the squeezing parameter λ defined as $\lambda = \tanh(r)$, where r is the squeezing constant. The figure shows that the Bell-CHSH inequality $|S| \le 2$ can be violated with the proposed setup and there is an optimal squeezing which maximizes S. The maximum Bell factor achievable with our scheme considering perfect detectors is $S_{\max} \approx 2.045$ which represents a violation of the Bell inequality by 2.2%. To get close to the S_{max} one needs to achieve $\lambda T \approx 0.57$ which corresponds to approximately 6 dB of squeezing for $T = 0.95$. Figures 2 (a),(b) illustrate that by increasing T we can enhance the Bell violation but only at the expense of reducing the probability of state generation P_G.

It follows from Fig. 2(c) that the Bell factor S is not very sernsitive to the efficiency of the single-photon detectors η_{PD}, so the Bell inequality can be violated even if $\eta_{\mathrm{PD}} \approx 1\%$. Low detection efficiencies only decrease the probability of conditional generation P_G of the non-Gaussian state. In contrast, the Bell factor S strongly depends on the efficiency of the homodyne detectors η_{BHD}, which must be above $\sim 90\%$ in order to observe Bell violation, see Fig. 2 (d). However, this does not represent a significant problem since such (and even higher) efficiencies have already been achieved experimentally (see *e.g.* [6]).

ALTERNATIVE SCHEMES

From the practical point of view, it would be most interesting to use a scheme that involves only one single-photon subtraction, because each photon subtraction is a rather complicated operation that moreover reduces the probability of state preparation. We have systematically studied schemes consisting of sources of squeezed light, photon subtraction, passive linear optics and balanced homodyning. We have not been able to find a scheme with only a single subtraction which would exhibit violation of Bell inequalities. Also we have not found any scheme with three subtractions that would exhibit violation of Bell inequalities. Since a scheme with four photon subtractions results only in a marginal improvement of the maximum Bell violation (increase from 2.2% to 3%), while the setup becomes much more complex [12], we can conclude that the setup proposed in references [10, 17] is the most appropriate to test Bell-CHSH inequalities using linear optics, photon subtraction, homodyne detection and sign binning.

CONCLUSIONS

The existence of an experimental window for a loophole-free Bell test seems plausible: with $\eta_{PD} = 30\%$, $T = 95\%$, and 6 dB squeezing, a violation of Bell inequalities by about 1% should be observable if the homodyne efficiency $\eta_{\rm BHD}$ is larger than $\sim 90\%$ and the electronic noise of the homodyne detection is at least 15 dB below the shot noise level. A pulse repetition rate in the range of 1 MHz as reported in Ref. [11] would provide enough time for individual pulse analysis and random bit generation. With this repetition rate and the above parameters, the number of data samples would be several hundreds per second, so the total time of the measurement necessary to collect enough data in order to reduce the statistical error below the percent range would be of the order of a few hours.

One of the main challenges on the way toward experimental implementation is to increase $\eta_{\rm PD}$, which was only about 1% in the recent experiment [11]. Another obstacle is that the photodetectors PD can be sometimes triggered by photons coming from other modes than the modes observed in homodyne detectors, because the OPA emits squeezed light into several spatio-temporal modes. These false triggers would degrade the quality of the conditionally generated state and would reduce the Bell factor S. The suppression of the false triggers and the increase of η_{PD} can be potentially achieved by improved spatial and spectral filtering.

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