Quantum cloning of a pair of orthogonally polarized photons with linear optics

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A linear optical probabilistic scheme for the optimal cloning of a pair of orthogonally polarized photons is devised, based on single- and two-photon interferences. It consists in a partial symmetrization device realized with a modified unbalanced Mach-Zehnder interferometer, followed by two balanced beam splitters where the Hong-Ou-Mandel photon bunching occurs. This scheme has the advantage that it enables quantum cloning without the need for stimulated amplification in a nonlinear medium. It can also be modified so to make an optical two-qubit partial SWAP gate, thereby providing a potentially useful tool to linear optics quantum computing.

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I. INTRODUCTION

Perfect copying of unknown quantum states is forbidden due to the linearity of quantum mechanics [1,2]. The rapid development of quantum information theory over the last decade has stimulated the investigation of optimal approximate quantum copying transformations, which produce two or more copies of a state with maximum fidelity [3,4]. The reason behind this interest is twofold. First, the optimal "quantum cloners" provide insight into the fundamental limits on the manipulation and distribution of quantum information. Second, from a more practical point of view, these cloners can be used as very efficient eavesdropping attacks on quantum key distribution protocols. Of particular interest is the cloning of single photons, which are ideal carriers of quantum information as they can be distributed over long distances through optical fibers or free space. In this context, the optimal universal copying of the polarization state of single photons has been thoroughly investigated theoretically and successfully demonstrated experimentally by several groups. These experiments can be divided, roughly speaking, into two classes. The first strategy, suggested in Ref. [5], consists in exploiting the quantum-noise limited amplification of light in a nonlinear crystal or in a fiber amplifier. It was demonstrated in Refs. [6–8]. The second strategy is to make the source photon interfere with an auxiliary photon prepared in a maximally mixed state on a beam splitter [9–12]. The bunching of photons then ensures the symmetrization of the total state of the photons, which is a way of effecting the optimal universal cloning transformation as shown in Ref. [13].

We thus observe that two fundamental quantum optical processes which are quite unrelated, namely the amplification of light and the multiphoton interference, become interchangeable as far as quantum cloning is concerned. In this paper, we further exploit this interesting relationship by designing an interferometric scheme for the optimal quantum copying of a pair of orthogonally polarized photons [14]. In the latter scenario, one assumes that the state to be cloned is formed by a pair of qubit states $|\psi\rangle|\psi_\perp\rangle$, where $\langle\psi|\psi_\perp\rangle=0$ and $|\psi\rangle$ can be arbitrary. The optimal cloning operation which produces M copies of the state $|\psi\rangle$ from the state $|\psi\rangle|\psi_\perp\rangle$ with highest fidelity was derived in Ref. [14], where it was shown that, surprisingly, the attained fidelity exceeds that of the cloning of a pair $|\psi\rangle|\psi\rangle$ when $M>6$. It was also proved that the optimal cloning of $|\psi\rangle|\psi_\perp\rangle$ could be probabilistically accomplished with type-II nondegenerate parametric down-conversion, similarly as for the cloning of a single photon. The trick is that the photon in polarization state $|\psi\rangle$ must be fed in the signal input port of the amplifier, while the photon in state $|\psi_\perp\rangle$ must be fed in the idler input port. With a certain probability, the amplifier produces M copies of $|\psi\rangle$ in the output signal port, with the value of M being determined by measuring the number of output idler photons. The distinct feature of this scheme is that the fidelity of the clones depends on the amplification gain, which must be set to the optimal value in order to recover the optimal cloning transformation [14].

In the present work, we show how to implement the universal cloning of the state $|\psi\rangle|\psi_\perp\rangle$ with passive linear optics and auxiliary photons, circumventing the need for active nonlinear media. The term "universal" means that the transformation is independent of the state $|\psi\rangle$, so that our scheme can be viewed as a way to effect the polarization-insensitive amplification of $|\psi\rangle|\psi_\perp\rangle$ without an optical amplifier. In Sec. III, we will explain the working of our proposed scheme for the simplest yet nontrivial example of the generation of two copies; we will then extend it to arbitrary M. A crucial part of our proposal happens to be the partial symmetrization of the two-photon polarization state. Therefore, in Sec. II, we will first show how the partial symmetrization device can be (probabilistically) accomplished by an interference of two photons in a specifically designed Mach-Zehnder interferometer. Remarkably, a simple modification of this device also enables the realization of a two-qubit optical partial-SWAP gate. Our scheme may thus find applications in various areas of quantum information processing with linear optics, beyond quantum cloning.

II. PARTIAL SYMMETRIZATION DEVICE

Let us begin with the description of the partial symmetrization device. Let Π_+ and Π_- denote projectors onto the symmetric and antisymmetric subspace of the polarization

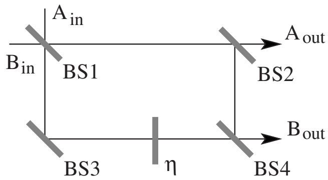

FIG. 1. Partial symmetrization of the polarization state of two photons. The scheme consists of four balanced beam splitters BSj and one attenuator η.

space of two photons. The nonunitary partial symmetrization operation is defined as

$$|\Psi\rangle_{AB} \mapsto (\Pi_+ + \eta\Pi_-)|\Psi\rangle_{AB}, \tag{1}$$

where $0 \leq \eta \leq 1$. The optical scheme which conditionally implements this transformation is shown in Fig. 1. It is essentially a Mach-Zehnder interferometer made of four balanced beam splitters BS1–BS4, with a variable attenuator of amplitude transmittance η placed in one of its arms. A single photon is injected into each input port A_{in} and B_{in}, and the partial symmetrization is successful if a single photon is present in each output port A_{out} and B_{out}. Note that these two output modes differ from those of a usual Mach-Zehnder interferometer since one output beam is obtained by tapping-off a part of the beam in the upper arm with the help of a balanced beam splitter BS2. The two input photons first interfere on the balanced beam splitter BS1, which forms a Hong-Ou-Mandel interferometer. If the polarization state of the two photons is symmetric then bunching occurs and both photons end up in the same arm of the interferometer. With probability 1/2, both photons follow the lower arm so that none can reach the output port A_{out}, hence the device fails. However, with probability 1/2, both photons are in the upper arm and then, again with probability 1/2, one photon is reflected at BS2 while the other is transmitted through BS2, ending in the output mode A_{out}. Finally, the reflected photon can be again reflected with probability 1/2 at BS4, ending in mode B_{out}. Taking all these probabilities into account, we conclude that the symmetric part of the input polarization state transforms according to

$$\Pi_+|\Psi\rangle_{AB} \mapsto \frac{1}{2\sqrt{2}}\Pi_+|\Psi\rangle_{AB}. \tag{2}$$

Now, if the two-photon input state is antisymmetric, then the photons are in the maximally entangled singlet state $|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|V\rangle_A|H\rangle_B - |H\rangle_A|V\rangle_B)$. The photons never bunch and, after interference at BS1, one photon is found in each arm of the interferometer so that the polarization state remains $|\Psi^-\rangle$. This well-known effect was exploited, e.g., in quantum teleportation to carry out a partial Bell state analysis, that is to discriminate the singlet from the other three Bell states [15]. In our scheme, the photon in the upper arm is transmitted with probability 1/2 through BS2 into mode A_{out}. In parallel, with the overall probability $\eta^2/4$, the photon in the lower arm is reflected at BS3, passes through the attenuator η and BS4, and reaches mode B_{out}. It follows that the antisymmetric part of the input polarization state transforms according to

$$\Pi_-|\Psi\rangle_{AB} \mapsto \frac{\eta}{2\sqrt{2}}\Pi_-|\Psi\rangle_{AB}. \tag{3}$$

If the interferometer is balanced such that the photons traveling through the upper and lower arms perfectly overlap at BS4, then the two above operations act coherently. Since an arbitrary input state can be decomposed into a symmetric and antisymmetric part, $|\Psi\rangle_{AB} = \Pi_+|\Psi\rangle_{AB} + \Pi_-|\Psi\rangle_{AB}$, the linearity of quantum mechanics implies that overall conditional transformation reads as

$$|\Psi\rangle_{AB} \mapsto \frac{1}{2\sqrt{2}}(\Pi_+ + \eta\Pi_-)|\Psi\rangle_{AB}, \tag{4}$$

which is proportional to the desired partial symmetrization operation (1). The probability of success generally depends on the input state and can be expressed as

$$P_{\mathrm{sym}} = \frac{1}{8}\langle\Psi|(\Pi_+ + \eta^2\Pi_-)|\Psi\rangle. \tag{5}$$

The fact that that the setup depicted in Fig. 1 effectively implements the partial symmetrization transformation (4) can also be verified by direct calculation in the Fock basis, where the input two-photon state is written with the action of bosonic creation operators onto the vacuum state, the creation operators of the input modes are replaced by appropriate linear combinations of the output creation operators, and finally only those terms are kept where a single photon is present in each output mode A_{out} and B_{out}. The resulting scheme is very versatile because the degree of symmetrization can be controlled simply by changing the attenuation η in one arm of the interferometer. The case with full attenuation corresponds to the full symmetrization operation, while the case with no attenuation simply effects the identity.

Interestingly, the success probability of the partial symmetrization can be increased by using unbalanced beam splitters with real amplitude transmittances t_j and reflectances r_j, where $j=1,2,3,4$, and $t_j^2 + r_j^2 = 1$. The attenuation in the lower arm can be absorbed into the reflectance of BS3 by substitution $r_3 \mapsto \eta r_3$, so the only free parameters are the t_j's and we assume that there is no attenuator in the scheme. One can express the probability amplitude that two photons in a symmetric or antisymmetric input state reach the desired output ports of the interferometer as $\alpha_{\mathrm{sym}} = 2t_1r_1t_2r_2r_4 + (r_1^2 - t_1^2)t_2r_3t_4$ and $\alpha_{\mathrm{antisym}} = t_2r_3t_4$, respectively. Then, one can show that the conditional transformation preserves the form $|\Psi\rangle_{AB} \mapsto \sqrt{\mathcal{P}}(\Pi_+ + \eta\Pi_-)|\Psi\rangle_{AB}$, provided that the condition $\eta\alpha_{\mathrm{sym}} = \alpha_{\mathrm{antisym}}$ holds. The success probability is proportional to the parameter $\mathcal{P}$ given by

$$\mathcal{P} = \frac{1}{\eta^2}t_2^2r_3^2t_4^2. \tag{6}$$

The maximization of $\mathcal{P}$ over all admissible values of t_j can be performed analytically and the optimal intensity transmittances $T_j = t_j^2$ read as $T_1 = (1-\eta)/2$, $T_2 = (1-\sqrt{1-\eta^2})/\eta^2$, $T_3 = 0$, and $T_4 = 1-\sqrt{1-\eta^2}$. This yields

$$\mathcal{P}_{\max} = \frac{1}{\eta^4}(1 - \sqrt{1 - \eta^2})^2. \tag{7}$$

Note that BS3 is actually replaced by a perfect mirror, which minimizes the leakage of photons into unwanted modes. The efficiency of the device is enhanced at the expense of the dependence of the T_j's on η. This means that, in contrast to the scheme with balanced beam splitters, the degree of symmetrization cannot be controlled simply by changing the attenuation in one arm of the interferometer. Instead, one must simultaneously tune the splitting ratios of BS1, BS2, and BS4. This makes this latter scheme less practical than the former as it would be difficult, in practice, to change the splitting ratios as required and it is much easier to control the degree of symmetrization with a variable attenuator.

In addition, it is also possible to increase $\mathcal{P}$ while using configurations where, as in the original scheme, the T_j's do not depend on η. The degree of symmetrization is again determined by an attenuator, as shown in Fig. 1. Specifically, if we set

$$T_1 = \frac{1}{2}, \quad T_2 = \frac{1-2T_4}{1-T_4}, \quad T_3 = 0, \tag{8}$$

then we have

$$\mathcal{P} = \frac{T_4(1-2T_4)}{1-T_4}. \tag{9}$$

This is maximized for $T_4 = 1 - 1/\sqrt{2}$, namely $\mathcal{P} = (\sqrt{2}-1)^2 \simeq 0.172 > 1/8$. Another interesting choice is to set $T_4 = 1/3$, resulting in $T_2 = 1/2$ and $\mathcal{P} = 1/6$. This last scheme thus requires two balanced beam splitters BS1 and BS2, one mirror BS3, a single unbalanced 2:1 beam splitter BS4, and a variable attenuator.

Note that, in all these schemes, the partial symmetrization relies on a fine interplay between single- and two-photon interferences. This is in contrast with the partial antisymmetrization device introduced in [16], which is defined in analogy with Eq. (1) but interchanging the roles of Π_+ and Π_-. In that case, the device works solely by a two-photon interference on an unbalanced beam splitter, whose reflectance determines the value of η (for a 50:50 beam splitter, we have full antisymmetrization). Our partial symmetrization device is thus experimentally more challenging, but it opens interesting new perspectives as we shall see.

III. OPTIMAL QUANTUM CLONING WITHOUT NONLINEARITIES

A. Generation of two clones of $|\psi\rangle|\psi_\perp\rangle$

Let us show how the above partial symmetrization device can be used in order to optimally clone a pair of orthogonal qubits $|\psi\rangle|\psi_\perp\rangle$ with linear optics. In our scheme, qubits are encoded into polarization states of single photons. We first consider the preparation of two clones, keeping the case of M clones for Sec. III C. Remember first how the polarization-insensitive cloning of single qubit $|\psi\rangle$ based on linear optics works. The input photon in state $|\psi\rangle$ impinges on a balanced beam splitter BS_1 where it interferes with an-

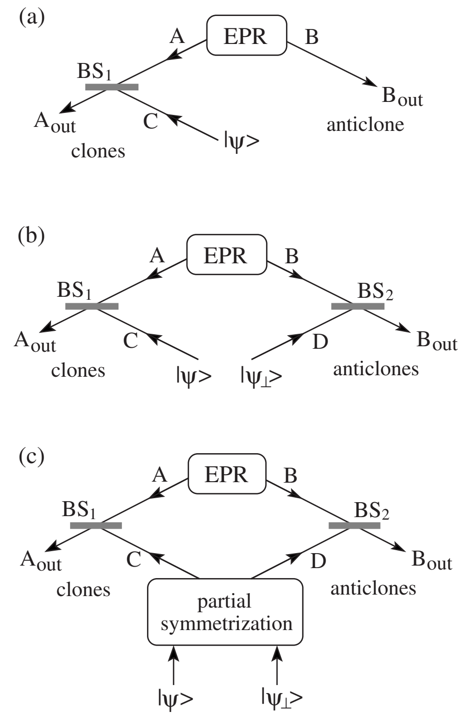

FIG. 2. (a) Optimal cloning of the polarization state $|\psi\rangle$ of one photon with linear optics. The EPR denotes an auxiliary two-photon polarization singlet state, while BS_1 is a balanced beam splitter. (b) Suboptimal cloning of a pair of orthogonally polarized photons with linear optics. The second input is prepared in the orthogonal polarization state $|\psi_\perp\rangle$, and BS_2 is another balanced beam splitter. (c) Optimal cloning of the pair $|\psi\rangle|\psi_\perp\rangle$ using the previous cloning scheme preceded by a partial symmetrization device.

other photon in a maximally mixed polarization state [see Fig. 2(a)]. The success of the cloning transformation is associated with the bunching of the two photons at the beam splitter BS_1, which heralds the symmetrization operation. Thus, the cloning is witnessed by the detection of two photons in the output mode A_{out} of BS_1. The state of the two clones of $|\psi\rangle$ is simply the polarization state of these two photons. Note that the maximally mixed state may be obtained by generating a two-photon polarization singlet state $|\Psi^-\rangle_{AB}$ [an Einstein-Podolsky-Rosen (EPR) pair], sending one photon of this pair on BS_1 [see Fig. 2(a)]. Interestingly, if we post-select the other photon of the pair in the cases where we have detection of the two clones in A_{out}, the polarization state of this photon in mode B then coincides with the so-called anticlone, i.e., the approximate version of $|\psi_\perp\rangle$.

Assume now that the second photon of the EPR pair (the one in mode B) impinges on another balanced beam splitter BS_2, where it interferes with mode D [see Fig. 2(b)]. If the latter mode is in the vacuum state and we only keep the cases where the photon emerges in mode B_{out}, the overall cloning transformation remains unchanged (except for a reduction of the success probability by a factor of 1/2). However, this modification is crucial because it introduces mode D in the scheme, which plays the same role as the input idler mode in the implementation of a cloner based on the amplification of light. When using parametric stimulated down-conversion in a nonlinear medium to effect cloning, one sends a photon in state $|\psi\rangle$ in the signal mode of the amplifier, and leaves the idler mode in the vacuum state. The two photons emerging in the signal mode are the clones, while the photon in the output idler mode encodes the anticlone. It was shown in Ref. [14] that the optimal cloning of a pair of orthogonally polarized photons $|\psi\rangle|\psi_\perp\rangle$ can be realized by injecting the photon in state $|\psi_\perp\rangle$ in the idler input mode instead of the vacuum. This improves the fidelity of the cloning of the state $|\psi\rangle$ that is sent into the signal input mode.

This suggests that a linear optics version of the cloning of $|\psi\rangle|\psi_\perp\rangle$ may be achieved similarly by injecting the photon in state $|\psi_\perp\rangle$ in mode D instead of the vacuum [see Fig. 2(b)]. The success of cloning is then heralded by the detection of two photons in mode $A_{\rm out}$ (the two clones) and two photons in mode $B_{\rm out}$ (the two anticlones). The resulting probabilistic scheme approximates quite well the unphysical transformation $|\psi\rangle|\psi_\perp\rangle \mapsto |\psi\rangle^{\otimes 2}|\psi_\perp\rangle^{\otimes 2}$. However, as we will see below, an additional element is needed in order to achieve the optimal cloning transformation, namely the partial symmetrization device described in Sec. II.

The final scheme, which realizes the optimal cloning, is illustrated in Fig. 2(c). The two-photon input state to be cloned $|\psi\rangle|\psi_\perp\rangle$ is first partially symmetrized using the setup of Fig. 1. The resulting state in modes C and D reads as

$$|\Psi_{\rm in}\rangle_{CD} = (\Pi_+ + \eta\Pi_-)|\psi\rangle_C|\psi_\perp\rangle_D = \frac{1+\eta}{2}|\psi\rangle_C|\psi_\perp\rangle_D + \frac{1-\eta}{2}|\psi_\perp\rangle_C|\psi\rangle_D, \quad (10)$$

where we made use of the relations $\Pi_+|\psi\rangle|\psi_\perp\rangle = (|\psi\rangle|\psi_\perp\rangle + |\psi_\perp\rangle|\psi\rangle)/2$ and $\Pi_-|\psi\rangle|\psi_\perp\rangle = (|\psi\rangle|\psi_\perp\rangle - |\psi_\perp\rangle|\psi\rangle)/2$. As in the previous scheme, this state then interferes on the two balanced beam splitters BS_1 and BS_2 with the auxiliary two-photon singlet state $|\Psi^-\rangle_{AB}$.

With the help of creation operators, we can express the states impinging on BS_1 and BS_2 as

$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(a_\psi^\dagger b_{\psi_\perp}^\dagger - a_{\psi_\perp}^\dagger b_\psi^\dagger)|0\rangle \quad (11)$$

and

$$|\Psi_{\rm in}\rangle_{CD} = \frac{1}{2}[(1+\eta)c_\psi^\dagger d_{\psi_\perp}^\dagger + (1-\eta)c_{\psi_\perp}^\dagger d_\psi^\dagger]|0\rangle, \quad (12)$$

where the subscript ψ ($\psi_\perp$) indicates creation operator for the polarization mode ψ ($\psi_\perp$). The cloning operation is probabilistic and succeeds if two photons are present in output mode $A_{\rm out}$ and two photons are present in $B_{\rm out}$. Indeed, in that case the state of photons in modes A and C (B and D) is thus symmetrized by the interference on BS_1 (BS_2). The output state can be determined from the input state $|\Psi^-\rangle_{AB}|\Psi_{\rm in}\rangle_{CD}$ by replacing the input creation operators with the appropriate linear combinations of output creation operators, namely

$$a_\psi^\dagger \mapsto \frac{1}{\sqrt{2}}(a_\psi^\dagger + c_\psi^\dagger), \quad b_\psi^\dagger \mapsto \frac{1}{\sqrt{2}}(b_\psi^\dagger + d_\psi^\dagger),$$

$$c_\psi^\dagger \mapsto \frac{1}{\sqrt{2}}(a_\psi^\dagger - c_\psi^\dagger), \quad d_\psi^\dagger \mapsto \frac{1}{\sqrt{2}}(b_\psi^\dagger - d_\psi^\dagger), \quad (13)$$

while similar relations hold for the creation operators for the $\psi_\perp$ polarization modes. By keeping only the terms of zeroth order in $c_\psi^\dagger$, $c_{\psi_\perp}^\dagger$, $d_\psi^\dagger$, and $d_{\psi_\perp}^\dagger$, we obtain the following expression for the (unnormalized) output state conditional on having two photons in mode $A_{\rm out}$ and two photons in mode $B_{\rm out}$:

$$|\Psi_{2,\rm out}\rangle \propto (a_\psi^\dagger b_{\psi_\perp}^\dagger - a_{\psi_\perp}^\dagger b_\psi^\dagger)(a_\psi^\dagger b_{\psi_\perp}^\dagger + q a_{\psi_\perp}^\dagger b_\psi^\dagger)|0\rangle, \quad (14)$$

where $q \equiv (1-\eta)/(1+\eta)$ has been introduced for the sake of simplicity. Omitting the subscripts ψ and $\psi^\dagger$, we define $|j,k\rangle$ as the double Fock state containing j photons in polarization state $|\psi\rangle$ and k photons in polarization state $|\psi_\perp\rangle$. In particular, $|\psi\rangle_A|\psi_\perp\rangle_B$ becomes $|1,0\rangle_A|0,1\rangle_B$. Then, the output state (14) can be expressed as a linear combination of double Fock states,

$$|\Psi_{2,\rm out}\rangle \propto 2|2,0\rangle_A|0,2\rangle_B + (q-1)|1,1\rangle_A|1,1\rangle_B - 2q|0,2\rangle_A|2,0\rangle_B. \quad (15)$$

The first term on the right-hand side of Eq. (15) corresponds to perfect cloning, with two photons in mode $A_{\rm out}$ emerging in the polarization state $|\psi\rangle$ and two photons in mode $B_{\rm out}$ being in state $|\psi_\perp\rangle$. In the second term, one photon has the right polarization and one photon has the wrong polarization in both $A_{\rm out}$ and $B_{\rm out}$, thus contributing to 1/2 in the cloning fidelity. The third term does not contribute to the cloning fidelity since the two photons have a polarization state that is orthogonal to the expected one in each output mode. Thus, the fidelity of single clones in mode $A_{\rm out}$ (or single anticlones in mode $B_{\rm out}$) can be evaluated as

$$F_\perp(2,q) = \frac{1\times 2^2 + \frac{1}{2}\times(q-1)^2}{2^2 + (q-1)^2 + (2q)^2} = \frac{q^2-2q+9}{2(5q^2-2q+5)}. \quad (16)$$

Note that in deriving formula (16), we have taken into account that the state (15) is not normalized.

Coming back to the simplified cloning scheme of Fig. 2(b), we see that removing the partial symmetrization device is equivalent to taking $\eta=1$ (or $q=0$) in the scheme of Fig. 2(c). In that case, the first term of Eq. (15) weighs 2^2 and the second term 1^2, while the third term vanishes. The resulting single-clone fidelity is 9/10. We notice that by choosing a small value of $q>0$, the weight of the second term decreases linearly with q while that of the third term increases only quadratically with q. Since these two terms correspond to cloning noise, it is clear that increasing q is advantageous as it decreases the overall noise up to some extent. The optimal q that maximizes $F_\perp(2,q)$ (or minimizes the noise) can be determined by solving the equation $\frac{\partial F_\perp(2,q)}{\partial q}=0$, which yields

$$q_{2,\rm opt} = 5 - 2\sqrt{6}. \quad (17)$$

By inserting $q_{2,\rm opt}$ back into Eq. (16), we obtain

$$F_{\perp}(2) = \frac{1}{2}\left(1 + \sqrt{\frac{2}{3}}\right) \simeq 0.908 \tag{18}$$

which is the maximum achievable fidelity [14]. Our scheme thus optimally clones the pair of orthogonal photonic qubits, as advertised. Note that the achieved fidelity is indeed slightly larger than that of the simplified scheme which is not preceded by the partial symmetrization of $|\psi\rangle|\psi_{\perp}\rangle$, namely 9/10. Note also that the optimal attenuation corresponding to the value of q of Eq. (17) reads as $\eta_{2,\mathrm{opt}}=\sqrt{2/3}$. The total probability of success of the cloning scheme is given by $P_{\mathrm{tot}}=P_{\mathrm{sym}}P_{\mathrm{EPR}}$, where $P_{\mathrm{sym}}=5/48$ is the probability of successful partial symmetrization (in the scheme with balanced beam splitters) and $P_{\mathrm{EPR}}=3/20$ is the probability of success of the scheme in Fig. 2(b). Hence, we have $P_{\mathrm{tot}}=1/64$.

B. Experimental feasibility of the proposed setup

Let us briefly discuss how the setup of Fig. 2(c) may be demonstrated experimentally. First note that although the partial symmetrization device operates on a coincidence basis, this does not negatively affect the rest of the scheme shown in Fig. 2(c). This is because after partial symmetrization, the two emerging photons are not recombined at any further beam splitter, but participate each in another separate two-photon interference. Thus, if we know that a two-photon singlet state was injected into input modes A and B, then the detection of two photons in modes A_{out} and B_{out} confirms the successful partial symmetrization. To prepare the two-photon input state one could utilize, for example, two single photons, each one being prepared conditionally from a photon pair generated via spontaneous parametric down-conversion by triggering on detections of the idler photons. The auxiliary singlet state in modes A and B could be prepared in the same way. The whole experiment would then involve a six-photon coincidence. Even if very challenging, recent experiments with three photon pairs and sixfold coincidence measurements were reported [17,18], so our proposed scheme is within the reach of current technology.

The partial symmetrization device requires interferometric stability. Recently, the interference of two photons in bulk Mach-Zehnder interferometer was demonstrated experimentally and explored for the optimal universal asymmetric cloning of single photons [19]. Moreover, the combination of single- and two-photon interferences has recently been utilized in fiber-based experiments, where very high visibility was achieved [20,21]. This suggests that our cloning scheme is experimentally realizable.

An interesting feature of our scheme is that the nonlinearity which is inherent to the cloner when realized with stimulated amplification is hidden here in the prior preparation of the EPR pair. This can be viewed as a nonlinear resource which is prepared "off-line," and used only later on when needed, which is reminiscent to the idea behind linear-optics quantum computing [22,23].

C. Generation of M clones of $|\psi\rangle|\psi_{\perp}\rangle$

Let us finally show how our scheme can be extended to more than two clones. The optimal copying operation which produces M copies of $|\psi\rangle$ from the state $|\psi\rangle|\psi_{\perp}\rangle$ can be written in a covariant form as follows [14]:

$$|\psi\rangle_A|\psi_{\perp}\rangle_B \mapsto \sum_{j=0}^{M} \alpha_{j,M}|(M-j)\psi,j\psi_{\perp}\rangle_A|j\psi,(M-j)\psi_{\perp}\rangle_B, \tag{19}$$

where the coefficients $\alpha_{j,M}$ can be expressed as

$$\alpha_{j,M} = \frac{(-1)^j}{\sqrt{2(M+1)}}\left(1 + \frac{\sqrt{3}(M-2j)}{\sqrt{M(M+2)}}\right) \tag{20}$$

and $|j\psi,(M-j)\psi_{\perp}\rangle$ denotes a totally symmetric polarization state of M photons in a single spatiotemporal mode, with j photons in polarization state $|\psi\rangle$ and $M-j$ photons in state $|\psi_{\perp}\rangle$. The transformation (19) achieves the fidelity

$$F_{\perp}(M) = \frac{1}{2}\left(1 + \sqrt{\frac{M+2}{3M}}\right). \tag{21}$$

A physical insight into the structure of the unitary transformation (19) is obtained by considering cloning via stimulated amplification. With a suitable choice of the crystal geometry, the type-II parametric down-conversion is governed by the Hamiltonian

$$H = ig(a_V^{\dagger}b_H^{\dagger} - a_H^{\dagger}b_V^{\dagger}) + \mathrm{H.c.}, \tag{22}$$

where $a_V^{\dagger}$ ($a_H^{\dagger}$) and $b_V^{\dagger}$ ($b_H^{\dagger}$) denote creation operators of the vertically (horizontally) polarized modes of the signal and idler beam, respectively, and g is the effective amplification gain, proportional to the amplitude of the pump beam, the second-order susceptibility of the crystal, and the crystal length. Since H is invariant under simultaneous identical local polarization rotation of both the signal and idler beams, $(U\otimes U)H(U^{\dagger}\otimes U^{\dagger})=H$, we can write

$$H = ig(a_{\psi}^{\dagger}b_{\psi_{\perp}}^{\dagger} - a_{\psi_{\perp}}^{\dagger}b_{\psi}^{\dagger}) + \mathrm{H.c.} \tag{23}$$

The output state of the amplifier reads as $|\Psi_{M,\mathrm{out}}\rangle = e^{-iH}|\psi\rangle_A|\psi_{\perp}\rangle_B$, where A stands for the signal mode and B for the idler mode. The unitary transformation can be conveniently written in a factorized form,

$$e^{-iH} = e^{\lambda X_{\psi}}(1-\lambda^2)^{n_{\mathrm{tot}}/2+1}e^{-\lambda X_{\psi}^{\dagger}}, \tag{24}$$

where $\lambda=\tanh(g)$, $X_{\psi}=a_{\psi}^{\dagger}b_{\psi_{\perp}}^{\dagger}-a_{\psi_{\perp}}^{\dagger}b_{\psi}^{\dagger}$, and n_{tot} denotes the total photon-number operator in all four modes, $n_{\mathrm{tot}}=a_{\psi}^{\dagger}a_{\psi}+a_{\psi_{\perp}}^{\dagger}a_{\psi_{\perp}}+b_{\psi}^{\dagger}b_{\psi}+b_{\psi_{\perp}}^{\dagger}b_{\psi_{\perp}}$. Using Eq. (24), it can be shown by a straightforward calculation that the output state corresponding to having M photons in each mode (signal and idler) reads as

$$|\Psi_{M,\mathrm{out}}\rangle \propto X_{\psi}^{M-1}(a_{\psi}^{\dagger}b_{\psi_{\perp}}^{\dagger} + qa_{\psi_{\perp}}^{\dagger}b_{\psi}^{\dagger})|0\rangle, \tag{25}$$

where the coefficient q depends on the gain g. For each value of M, there is an optimal q which maximizes the cloning fidelity, $q_{M,\mathrm{opt}}=(\sqrt{3M}-\sqrt{M+2})/(\sqrt{3M}+\sqrt{M+2})$.

Finally, comparing Eq. (25) with Eq. (14) suggests that the state $|\Psi_{M,\mathrm{out}}\rangle$ can be similarly prepared using the scheme shown in Fig. 2(c), where the input state is partially symmetrized with factor $\eta=(1-q)/(1+q)$ while the EPR state is replaced by the $2(M-1)$-photon state produced by spontaneous type-II parametric down-conversion, $|\Psi_{\mathrm{EPR}}\rangle \propto X_{\psi}^{M-1}|0\rangle$. Note that this state, corresponding to $(M-1)$ photon pairs, can be generated "off-line" in a nonlinear crystal with low gain g, as in the case of $M=2$. In contrast, the implementation of this cloner with stimulated amplification requires a gain g which grows with M. Interestingly, we thus conclude that the above linear optical scheme enables us to simulate the polarization-insensitive amplification of $|\psi\rangle|\psi_{\perp}\rangle$ with arbitrarily high gain using only a "off-line" low-gain nonlinear process, albeit with a low success probability.

IV. CONCLUSIONS

It was shown that the optimal quantum cloning of a pair of photons with orthogonal polarizations $|\psi\rangle|\psi_{\perp}\rangle$ can be experimentally realized by using a linear optical scheme, avoiding the traditional use of stimulated parametric down-conversion in a nonlinear crystal. An appropriate combination of one- and two-photon interferences makes it possible to effect, with some success probability, the transformation resulting into M clones and M anticlones from the pair $|\psi\rangle|\psi_{\perp}\rangle$ and an auxiliary $2(M-1)$ photon entangled state $|\Psi_{\mathrm{EPR}}\rangle$. This transformation consists in the sequence of a partial symmetrization device, acting on the input state $|\psi\rangle|\psi_{\perp}\rangle$, followed by two balanced beam splitters where the Hong-Ou-Mandel bunching effect is used as a means to symmetrize the state.

The partial symmetrization device is another probabilistic interferometric scheme, with an attenuator η placed in one of the interferometer arms in order to tune the symmetrization parameter. Moreover, if this attenuator is replaced by a phase shifter, it appears that the symmetrization scheme instead effects the unitary partial SWAP gate [with probability 1/8 or $(\sqrt{2}-1)^2$ if unbalanced beam splitters are employed]. Formally, we can substitute $\eta=e^{i\phi}$ in Eq. (1), and get the unitary transformation

$$U(\phi) = \Pi_{+} + e^{i\phi}\Pi_{-}. \tag{26}$$

For $\phi=\pi$, we recover the SWAP gate, which interchanges the states of the two photons. Of particular importance is the square-root SWAP gate, which is achieved by choosing $\phi=\pi/2$. This gate, together with arbitrary single-qubit polarization rotations, is sufficient for universal quantum computing. Thus, we have found a whole new class of two-qubit optical quantum gates to be inserted in the toolbox of available linear-optics quantum gates [22,23]. The gate also allows us to investigate quantum decoherence in the process of quantum homogenization, where a qubit is successively coupled to auxiliary qubits via partial SWAP gates [24,25]. In this way it is possible to simulate relaxation of the qubit toward equilibrium. The partial SWAP gate for polarization states of single photons has been very recently successfully demonstrated experimentally [26], which clearly confirms the experimental feasibility of the present proposal.

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[1] W. K. Wootters and W. H. Zurek, Nature (London) **299**, 802 (1982).
[2] D. Dieks, Phys. Lett. **92A**, 271 (1982).
[3] N. J. Cerf and J. Fiurášek, in *Progress in Optics*, edited by E. Wolf (Elsevier, New York, 2006), Vol. 49, p. 455.
[4] V. Scarani, S. Iblisdir, N. Gisin, and A. Acín, Rev. Mod. Phys. **77**, 1225 (2005).
[5] C. Simon, G. Weihs, and A. Zeilinger, Phys. Rev. Lett. **84**, 2993 (2000).
[6] A. Lamas-Linares, C. Simon, J. C. Howell, and D. Bouwmeester, Science **296**, 712 (2002).
[7] F. De Martini, D. Pelliccia, and F. Sciarrino, Phys. Rev. Lett. **92**, 067901 (2004).
[8] S. Fasel, N. Gisin, G. Ribordy, V. Scarani, and H. Zbinden, Phys. Rev. Lett. **89**, 107901 (2002).
[9] M. Ricci, F. Sciarrino, C. Sias, and F. De Martini, Phys. Rev. Lett. **92**, 047901 (2004).
[10] W. T. M. Irvine, A. L. Linares, M. J. A. de Dood, and D. Bouwmeester, Phys. Rev. Lett. **92**, 047902 (2004).
[11] I. Ali Khan and J. C. Howell, Phys. Rev. A **70**, 010303(R) (2004).
[12] L. Masullo, M. Ricci, and F. De Martini, Phys. Rev. A **72**, 060304(R) (2005).
[13] R. F. Werner, Phys. Rev. A **58**, 1827 (1998).
[14] J. Fiurášek, S. Iblisdir, S. Massar, and N. J. Cerf, Phys. Rev. A **65**, 040302(R) (2002).
[15] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).
[16] R. Filip, Phys. Rev. A **69**, 052301 (2004).
[17] Q. Zhang, A. Goebel, C. Wagenknecht, Y.-A. Chen, B. Zhao, T. Yang, A. Mair, J. Schmiedmayer, and J.-W. Pan, Nat. Phys. **2**, 678 (2006).
[18] C.-Y. Lu, X.-Q. Zhou, O. Gühne, W.-B. Gao, J. Zhang, Z.-S. Yuan, A. Goebel, T. Yang, and J.-W. Pan, Nat. Phys. **3**, 91 (2007).
[19] Z. Zhao, A.-N. Zhang, X.-Q. Zhou, Y.-A. Chen, C.-Y. Lu, A. Karlsson, and J.-W. Pan, Phys. Rev. Lett. **95**, 030502 (2005).
[20] L. Bartušková, A. Černoch, R. Filip, J. Fiurášek, J. Soubusta, and M. Dušek, Phys. Rev. A **74**, 022325 (2006).
[21] L. Bartušková, M. Dušek, A. Černoch, J. Soubusta, and J. Fiurášek, Phys. Rev. Lett. **99**, 120505 (2007).
[22] E. Knill, R. Laflamme, and G. J. Milburn, Nature (London)

409, 46 (2001).
[23] P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, Rev. Mod. Phys. **79**, 135 (2007).
[24] V. Scarani, M. Ziman, P. Štelmachovič, N. Gisin, and V. Bužek, Phys. Rev. Lett. **88**, 097905 (2002).
[25] M. Ziman, P. Štelmachovič, V. Bužek, M. Hillery, V. Scarani, and N. Gisin, Phys. Rev. A **65**, 042105 (2002).
[26] A. Černoch, J. Soubusta, L. Bartušková, M. Dušek, and J. Fiurášek, Phys. Rev. Lett. **100**, 180501 (2008).