Noiseless Loss Suppression in Quantum Optical Communication

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We propose a protocol for conditional suppression of losses in direct quantum state transmission over a lossy quantum channel. The method works by noiselessly attenuating the input state prior to transmission through a lossy channel followed by noiseless amplification of the output state. The procedure does not add any noise; hence, it keeps quantum coherence. We experimentally demonstrate it in the subspace spanned by vacuum and single-photon states, and consider its general applicability.

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Quantum communication holds the promise of unconditionally secure information transmission [1]. However, the distance over which quantum states of light can be distributed without significant disturbance is limited due to unavoidable losses and noise in optical links. Losses, as well as errors or decoherence, may in principle be overcome by the sophisticated techniques of quantum error correction [2–4], entanglement distillation [5–7], and quantum repeaters [8,9]. However, these techniques typically require encoding information into complex multimode entangled states, processing many copies of such states, and—even more challenging—using quantum memories [10,11]. In stark contrast to the situation for classical communication, losses in quantum communication cannot be compensated by amplifying the signal because the laws of quantum mechanics imply that any deterministic phase-insensitive signal amplification is unavoidably accompanied by the addition of noise [12].

Very recently, however, heralded noiseless amplification of light [13] was proposed as a way out, relaxing the deterministic requirement. The noiseless amplification is formally described by a quantum filter $g^n$, where $n$ is the photon number operator and $g > 1$ denotes the amplification gain. The noiseless amplifier thus modulates the amplitudes of Fock states $|n\rangle$ by factor $g^n$. This filtering can conditionally increase the amplitude of a coherent state $|\alpha\rangle$ without adding any noise, $g^n|\alpha\rangle \propto |g\alpha\rangle$. Although this cannot be done perfectly because $g^n$ is unbounded, faithful noiseless amplification is possible in any finite subspace spanned by the Fock states $|n\rangle$ with $n \leq N$, albeit with a correspondingly low probability scaling as $g^{-2N}$ in the worst case of input vacuum state. With current technology, it has been proven possible to faithfully noiselessly amplify states containing mostly vacuum and single-photon contributions [14–19].

The noiseless amplifier can improve the performance of quantum key distribution protocols [20–23] and it can also be used to distribute high-quality entanglement over a lossy channel [13,24]. Beyond that, the noiseless amplifier is not useful to suppress losses in direct transmission of arbitrary quantum states because it is not the inverse map of a lossy channel $L$. Any superposition of Fock states that is not a coherent state is mapped by $L$ onto a mixed state, and this added noise cannot be eliminated by noiseless amplification.

Here, we find a solution to this fundamental problem by exploiting the so-called noiseless attenuation, which can be viewed as a heralded but reversible type of loss in the sense that the state becomes closer to vacuum, while its purity and quantum coherence are preserved. Mathematically, the noiseless attenuator is described by an operator $\nu^n$ with $\nu < 1$. This filtering can be accomplished with the help of a beam splitter with amplitude transmittance $\nu < 1$ and a single-photon detector monitoring the auxiliary output port of the beam splitter, see Fig. 1(a). If the detector does not register any photon, then the amplitudes of Fock states $|n\rangle$ are attenuated according to $|n\rangle \mapsto \nu^n|n\rangle$. The noiseless attenuator transforms $|\alpha\rangle \mapsto |\nu\alpha\rangle$ and it is the proper inverse map of the noiseless amplifier $g^n$ with $g = 1/\nu$.

In this Letter, we prove that a suitable combination of noiseless attenuation and amplification provides a powerful tool to conditionally suppress losses in channel $L$ to an arbitrary extent without adding noise. Our scheme works as

![FIG. 1 (color online).](a) Implementation of noiseless attenuation with a beam splitter of amplitude transmittance $\nu$ and a single-photon detector, conditioning on projecting onto vacuum $|0\rangle$. (b) Conditional noiseless loss suppression in direct state transmission over a lossy channel $L$ by a combination of noiseless attenuation with transmittance $\nu$ and noiseless amplification with gain $g$.]

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shown in Fig. 1(b). Before transmission through $\mathcal{L}$, the input state is noiselessly attenuated with transmittance $\nu$. Intriguingly, this input-state preprocessing has the effect of preferentially reducing the weight of the Fock states that have a higher chance of being subsequently affected by losses in $\mathcal{L}$. After transmission through $\mathcal{L}$, the state is noiselessly amplified with gain $g = 1/(\nu \tau)$, where $\tau$ is the amplitude transmittance of $\mathcal{L}$. In the limit $\nu \rightarrow 0$, this procedure conditionally converts the lossy channel $\mathcal{L}$ into a perfect lossless channel on the subspace where noiseless amplification $|n\rangle \rightarrow g^n|n\rangle$ is faithfully performed.

In order to provide more insight into our protocol, let us consider the simple, yet important case of an input state formed by a superposition of vacuum and single-photon states, $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$. At the output of $\mathcal{L}$, we get the mixed state

$$\rho_{\text{loss}} = |\tilde{\psi}\rangle\langle\tilde{\psi}| + (1 - \tau^2)|c_1|^2|0\rangle\langle0|,$$

where $|\tilde{\psi}\rangle = c_0|0\rangle + \tau c_1|1\rangle$. A naive compensation of losses by noiseless amplification of $\rho_{\text{loss}}$ with gain $g = 1/\tau$ results in the transformation $|0\rangle \rightarrow |0\rangle$, $|1\rangle \rightarrow g|1\rangle$, and yields the state

$$\rho_{\text{amp}} \propto |\psi\rangle\langle\psi| + (1 - \tau^2)|c_1|^2|0\rangle\langle0|.$$

Note that there remains an extra vacuum noise term proportional to $(1 - \tau^2)|c_1|^2$. This noise term could be further suppressed by amplification with gain higher than $1/\tau$, but such an approach would over-amplify the single-photon contribution.

The right solution is to preprocess the state via noiseless attenuation before the lossy channel $\mathcal{L}$. The effective input state of $\mathcal{L}$ then becomes $|\psi_{\text{att}}\rangle = c_0|0\rangle + \nu c_1|1\rangle$, and the output state after attenuation, transmission, and amplification with $g = 1/(\nu \tau)$ reads

$$\rho_{\text{out}} \propto |\psi\rangle\langle\psi| + (1 - \tau^2)\nu^2|c_1|^2|0\rangle\langle0|.$$

This has reduced the unwanted vacuum noise term by a factor of $\nu^2$. In the limit $\nu \rightarrow 0$, this term vanishes and the output state becomes equal to the input pure state $|\psi\rangle$.

Our protocol formally resembles the scheme for the suppression of qubit decoherence due to zero-temperature energy relaxation by using partial quantum measurements [25,26], but, importantly, it compensates losses instead of qubit decoherence and can be extended to arbitrarily large Hilbert space as we show now. The lossy channel $\mathcal{L}$ with inputs restricted to the subspace spanned by Fock states $|n\rangle$ with $n \leq N$ can be described by a finite number $N + 1$ of Kraus operators $A_j$,

$$\rho_{\text{out}} = \mathcal{L}(\rho_{\text{in}}) = \sum_{j=0}^{N} A_j \rho_{\text{in}} A_j^\dagger,$$

where $A_j = \sum_{m=-j}^{N-j} \sqrt{\frac{(m+j)! (1 - \tau^2)^j/2 \tau^m}{m!}} |m\rangle\langle m + j|$ accounts for loss of $j$ photons in a channel. Assuming that the noiseless amplification is performed perfectly on the subspace of the first $N + 1$ Fock states by filter $G_N(g) = g^{-N} \sum_{n=0}^{N} g^n|n\rangle\langle n|$, the effective channel $\mathcal{M}$ formed by sequence of noiseless attenuation, losses, and noiseless amplification reads,

$$\mathcal{M}(\rho_{\text{in}}) = \sum_{j=0}^{N} G_N(g)A_j \rho_{\text{in}} A_j^\dagger G_N(g),$$

with $g = 1/(\nu \tau)$. Due to the structure of Kraus operators we find that $G_N(g)A_j \rho_{\text{in}} = g^{-N} \rho_{\text{in}} A_j^\dagger A_j$ and the effective channel can be expressed as,

$$\mathcal{M}(\rho_{\text{in}}) = g^{-2N} \rho_{\text{in}} + g^{-2N} \sum_{j=1}^{N} \nu^2 |B_j^\dagger \rho_{\text{in}} B_j|,$$

where $B_j = A_0^\dagger A_j$, and the inverse $A_0^\dagger = \sum_{n=0}^{N} \tau^{-n}|n\rangle\langle n|$ exists on the considered finite dimensional subspace. We can see that the combination of noiseless attenuation and amplification progressively suppresses $j$-photon losses by a factor of $\nu^2j$ and in the limit $\nu \rightarrow 0$ the lossy channel is converted into the identity channel, $\mathcal{M} \rightarrow I$. The success probability of the protocol is state dependent,

$$P_{\text{succ}} = g^{-2N} + g^{-2N} \sum_{j=1}^{N} \nu^2 \text{Tr}[B_j^\dagger \rho_{\text{in}} B_j].$$

and $P_{\text{succ}}$ is lower bounded by $g^{-2N}$.

We have experimentally demonstrated this protocol for superpositions of vacuum and single-photon states. In the experimental setup, shown in Fig. 2, a correlated photon pair is generated [27], and the signal photon serves as a probe of the lossy channel $\mathcal{L}$ while the idler photon drives noiseless amplification. We characterize the loss suppression mechanism by using the Choi-Jamiolkowski isomorphism [28,29] between quantum channels and bipartite states. We exploit the polarization degree of freedom of the signal photon. The vertically ($V$) polarized mode is transmitted through $\mathcal{L}$, while the horizontally ($H$) polarized mode is transmitted through a reference identity channel $I$. If the signal photon is initially diagonally polarized, $|\Psi^\rangle = \frac{1}{\sqrt{2}} (|1\rangle_v|0\rangle_H + |0\rangle_v|1\rangle_H)$, then we obtain at the output a two-mode state $\chi_{\mathcal{L}} = [\mathcal{L}_V \otimes I_H](|\Psi^\rangle)$ that is isomorphic to the channel $\mathcal{L}$,

$$\chi_{\mathcal{L}} = \frac{1 - \tau^2}{2} |00\rangle\langle00| + \frac{1}{2} (|01\rangle + \tau |10\rangle)(|01\rangle + \tau |10\rangle),$$

(8)

where the subscripts $H$ and $V$ were suppressed for simplicity. Although $\chi_{\mathcal{L}}$ is a two-qubit density matrix, its support is restricted to a three-dimensional subspace spanned by $|00\rangle$, $|10\rangle$, and $|01\rangle$. The state $|11\rangle$ is absent because no photons are generated in the passive channel $\mathcal{L}$. This property holds even if the channel is combined with noiseless attenuation and amplification because these operations only modulate Fock state amplitudes. In the tomographic reconstruction of $\chi_{\mathcal{L}}$ we can, therefore, restrict
Amplification gain of this scheme is given by noiseless attenuation that eliminates negative effects of limited detection efficiency from the experiment.

In Fig. 3 we plot the reconstructed $\chi_L$ for four different channels. Panel (a) shows $\chi_L$ when no losses are inserted in the path of vertically polarized signal photons. The reconstructed operator is very close to the identity-channel matrix $\chi_I = \ket{\Psi^+}\bra{\Psi^+}$. The similarity of a channel $L$ with respect to the identity channel $I$ is measured by the channel fidelity $F = \bra{\Psi^+}1\bra{\Psi^+}$, and we obtain $F = 0.958 \pm 0.002$. Then, the $\chi_L$ matrix of a channel $L$ with

\[
\begin{align*}
\chi_L &= \begin{pmatrix}
0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \\
1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\
0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \\
1/\sqrt{2} & 0 & 1/\sqrt{2} & 0
\end{pmatrix}
\end{align*}
\]

of channel characterization, the experiment can be interpreted as a faithful transfer of one part of a path-entangled single-photon state $\ket{\Psi^+}$ over a lossy channel [13]. A weakly entangled state $\ket{\Psi_+}$ is prepared at the input and the noiseless amplification increases entanglement of the output state while preserving its high purity [24].

The state analysis block including detectors $D_3$ and $D_4$ serves for a full tomographic analysis of the polarization state of the output signal photons. Moreover, we employ detector $D_1$ to monitor the fraction of photons that are lost in channel $L$. Note that, in contrast to noise-reduction schemes based on measurements of the environment [31], detector $D_1$ is not needed for the protocol itself and only serves here for channel characterization. We measure the two-photon coincidences $D_1 \& D_2$, $D_3 \& D_2$, and $D_4 \& D_2$ for different settings of the wave plates in the output analysis block, and from these data we completely determine $\chi_L$. This coincidence measurement amounts to post-selection that eliminates negative effects of limited detection efficiency from the experiment.

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\end{pmatrix}
\end{align*}
\]
50% losses ($\tau = 1/\sqrt{2}$) is depicted in Fig. 3(b). Losses introduce imbalance between the amplitudes of $|01\rangle$ and $|10\rangle$ states, and give rise to a nonzero probability for the $|00\rangle$ state, which represents the fraction of lost photons in the channel. If we attempt to compensate these losses by noiseless amplification with gain $g = \sqrt{2}$, we obtain the channel in Fig. 3(c). The amplification balances the amplitudes of $|01\rangle$ and $|10\rangle$ states, but there remains some population in the $|00\rangle$ state. This unwanted noise can be further suppressed if we include noiseless attenuation. The resulting channel for $\nu = 1/\sqrt{2}$ and $g = 2$ is shown in Fig. 3(d). In contrast with Fig. 3(c), the noise term is reduced while quantum coherence is preserved, as witnessed by the off-diagonal terms in the subspace spanned by $|01\rangle$ and $|10\rangle$.

We have systematically investigated the performance of the protocol as a function of the amplification gain $g$ for three different levels of losses: 25%, 50%, and 75%. The fidelity $F$ of the resulting quantum channel, plotted in Fig. 4(a), monotonically grows with $g$, and theory (solid lines) predicts $F \rightarrow 1$ in the high gain limit. The experimentally observed $F$ saturates at values slightly below 1, which can be attributed mainly to imperfect two-photon interference on PBS$_2$. The measured visibility of the Hong-Ou-Mandel dip [32] $V = 0.947 \pm 0.002$ is in good agreement with the observed saturation. For comparison, we also plot data for the naive loss compensation strategy based solely on noiseless amplification without input state preprocessing ($\nu = 1$). The results shown as diamonds demonstrate the fundamental limitation of this strategy. With increasing gain, the channel fidelity reaches the maximum $F_{\text{max}} = (3 - \tau^2)/(4 - 2\tau^2)$ for $g_{\text{opt}} = (2 - \tau^2)/\tau$, and then drops down due to over-amplification of the single-photon part of the state. We define an effective channel transmittance $T_{\text{eff}}$ as the conditional probability that a photon injected into the channel emerges at the output. Figure 4(b) demonstrates that $T_{\text{eff}}$ monotonically increases with $g$ and approaches unity in the high gain limit.

The noiseless loss suppression is a conditional operation; therefore, its success probability is a crucial parameter. Assuming pure input state $c_0|0\rangle + c_1|1\rangle$ we obtain

$$P_{\text{succ}} = |c_0|^2 g^{-2} + |c_1|^2 [\tau^2 + (1 - \tau^2)g^{-2}] ,$$

(9)

which agrees with Eq. (7) if $g = 1/(\nu\tau)$. The actual experimental success probability is significantly lower because it is reduced by imperfect collection efficiency $\eta_{\text{c}}$ of the idler photon and low overall detection efficiency $\eta_D$ of the heralding detector $D_2$. We can only roughly estimate $\eta_{\text{c}}\eta_D \approx 0.1$. On the other hand, by taking the ratio of the measured total coincidence rates for a given $\tau$ and $g$ and for the identity channel with $\tau = 1$, we can reliably estimate a relative success probability normalized such that $P_{\text{opt}} = 1$ for the identity channel. We expect this relative success probability to be equal to (9). Relative success probability for the probe state $|\Psi^+\rangle$ is plotted in Fig. 4(c). It is in good agreement with theoretical predictions obtained by setting $|c_0|^2 = |c_1|^2 = 1/2$ in Eq. (9) and it scales as $g^{-2}$ as expected.

The post-selection utilized in the present experiment could be avoided at the cost of increased technological complexity. Using photon addition [33] and subtraction [34–36], the noiseless attenuator and amplifier can both be accomplished with high fidelity in a heralded manner even with imperfect single-photon detectors [17,37,38]. In this way, the output state can be made fully available for further processing. By coherently combining addition and subtraction of $N$ photons, the noiseless amplifier or attenuator can be exactly implemented on the subspace of states containing up to $N$ photons. Note that even with this robust approach, highly efficient detectors would be required to reach acceptable success probability for $N > 1$.

In summary, we have experimentally demonstrated a protocol for conditional noiseless suppression of losses in quantum optical channels using quantum filters at the input and output of the channel. The procedure is universally applicable and can enable faithful transmission of fragile highly nonclassical or entangled states of light over lossy channels. We anticipate numerous potential applications of...
the present scheme in quantum communication, quantum metrology, and other fields where loss reduction is essential for optimum performance.

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