Information transmission in bosonic memory channels using Gaussian matrix-product states as near-optimal symbols

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Abstract. We seek for a realistic implementation of multimode Gaussian entangled states that can realize the optimal encoding for quantum bosonic Gaussian channels with memory. For a Gaussian channel with classical additive Markovian correlated noise and a lossy channel with non-Markovian correlated noise, we demonstrate the usefulness using Gaussian matrix-product states (GMPS). These states can be generated sequentially, and may, in principle, approximate well any Gaussian state. We show that we can achieve up to 99.9% of the classical Gaussian capacity with GMPS requiring squeezing parameters that are reachable with current technology. This may offer a way towards an experimental realization.

Keywords: Quantum communication, channel capacity, entanglement generation
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A central problem of information theory is to determine the maximal rate of information transmission via communication channels. The achievable upper bound on the classical information transmission rates via quantum channels is called classical capacity. Recently, much interest has been attracted by quantum channels with *memory*. The reason is that the classical information transmission rate for these channels may be increased by encoding information into entangled states. A widely used channel memory model is given by correlated noise over subsequent uses of the channel [1, 2, 3].

Among quantum channels, Gaussian bosonic channels have a particular interest because they rather well model common physical links, such as optical fibers or optical communication channels in free space. However, finding the classical capacity of an arbitrary quantum Gaussian channel is a difficult problem. A lower bound on the classical capacity of bosonic Gaussian channels is obtained with the help of optimization schemes restricted to Gaussian encodings. This quantity is called *Gaussian capacity*. The Gaussian encoding corresponds to the modulation of displacements in phase space of a Gaussian input state with covariance matrix (CM) γ_{in} according to a multi-variate Gaussian distribution with CM γ_{mod}.

The Gaussian capacity was studied in detail for the Gaussian channel with additive correlated noise [2] and the lossy channel with a non-Markovian correlated noise [3]. The action of Gaussian channels on the CM of the input states reads $\gamma_{\text{out}} = \kappa \gamma_{\text{in}} + \kappa' \gamma_{\text{env}}$, $\overline{\gamma} = \gamma_{\text{out}} + \kappa \gamma_{\text{mod}}$, where γ_{env} is the CM of the noise added by the channel, γ_{out} and $\overline{\gamma}$ are the CM of the output and modulated output state, respectively. The parameters of the transformation are $\kappa = \kappa' = 1$ for the channel with additive noise, and $\kappa = \eta$, $\kappa' = 1 - \eta$ for the lossy channel with beamsplitter transmittance $\eta \in [0, 1]$. The input

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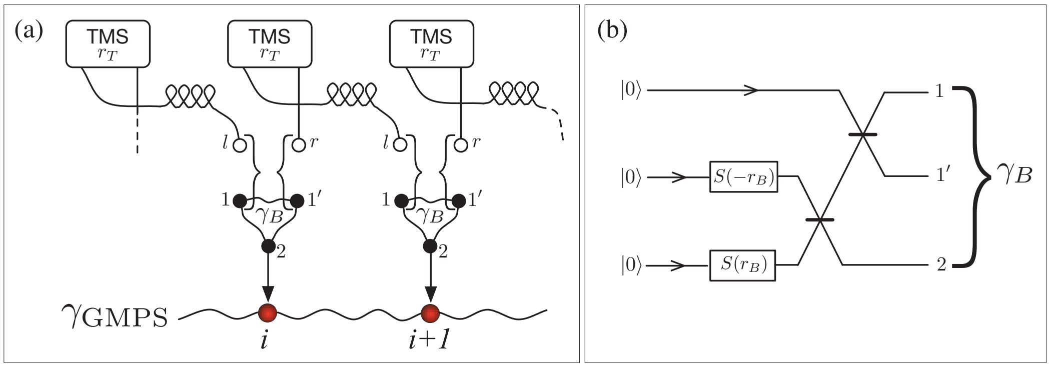

FIGURE 1. (a) The optical scheme suggested in [5] modified for the sequential generation of a GMPS with nearest-neighbor correlations. Two mode squeezed vacuum states (TMS) with squeezing parameter r_T are used for entangling subsequent channel modes with the help of a delay line. After two EPR measurements (represented by curly brackets) involving the modes l and r of two different TMS and the modes 1 and 1′ of an auxiliary three-mode building block γ_B, the mode 2 collapses into the GMPS mode i. (b) Optical setup for the generation of the three-mode building block γ_B used in (a). Here $|0\rangle$ denotes a vacuum mode, $S(r_B)$ is a one mode squeezer with squeezing parameter r_B and the bold horizontal lines represent 50 : 50 beamsplitters.

ensembles have to obey an energy constraint which, for optical modes of the same frequency, may be expressed in terms of the mean photon number $\bar{n}$ of the modulated input : $\frac{1}{2n}\mathrm{Tr}(\gamma_{\rm in}+\gamma_{\rm mod})-\frac{1}{2}=\bar{n}$. The noise CM is characterized by a pre-factor N, the correlation parameter $\phi\in[0,1]$ for the Markovian correlated noise, and the parameter $s\in\mathbb{R}$ for the non-Markovian correlated noise. The Gaussian capacity in this formulation is invariant under rotations in the phase space. If a rotation is found which *unravels* the noise correlations, the channel in the new basis becomes a collection of n uncorrelated single mode phase-sensitive Gaussian channels. Under the assumption of additivity the optimal encoding is given by a *quantum water-filling* [2, 3]. This solution goes beyond the water-filling solution for classical Gaussian channels, because in the quantum case one has to take into account an additional energy cost for the creation of optimal quantum input states which may be squeezed. We derived an input energy threshold, above which the quantum water-filling solution holds and we restrict here our study to energies above this threshold. In the original basis, the obtained optimal input states are in general entangled. Therefore finding experimental means for the generation of such states is a very challenging task.

As a candidate, we have chosen Gaussian matrix-product states (GMPS) [4, 5], which have a known optical implementation (Fig. 1) and can be created sequentially. The GMPS has a CM that can be written in terms of a circulant matrix $\mathscr{C}$ as $\gamma_{\rm GMPS}=\frac{1}{2}(\mathscr{C}^{-1}\oplus\mathscr{C})$. We determined the parameters of $\mathscr{C}$ achieving the highest transmission rate [6]. In a wide range of noise parameters, the GMPS can achieve more than 99.9% of the Gaussian capacity (see Fig. 2). The optical implementation of the GMPS requires squeezing operations, but, the required squeezing strength is achievable within present technology [7].

We have identified a class of channel noise models for which the GMPS is the exact

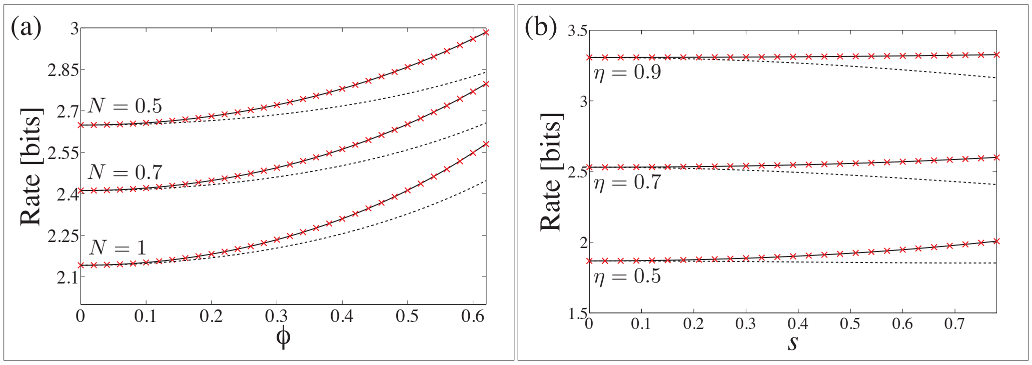

FIGURE 2. Transmission rates vs. correlation parameters ϕ and s for $\bar{n} = 5$. Plots of the Gaussian capacity (solid line), the rate using the GMPS ($\times$ markers) and the rate using coherent states (dotted line). (a) Channel with additive Markov noise, from top to bottom $N = \{0.5, 0.7, 1\}$. (b) Lossy channel with non-Markovian noise, for $N = 1$ and from bottom to top $\eta = \{0.5, 0.7, 0.9\}$.

optimal input state. Its CM is given by a circulant matrix $\mathscr{C}$ and an arbitrary matrix $\mathscr{N}_{\text{env}}$ which commutes with $\mathscr{C}$, i.e., $\gamma_{\text{env}} = (\mathscr{N}_{\text{env}} \oplus \mathscr{N}_{\text{env}})\,\frac{1}{2}(\mathscr{C}^{-1} \oplus \mathscr{C})$. We emphasize the fact that GMPS are known to be ground states of particular quadratic Hamiltonians of harmonic lattices. In particular, the GMPS is the ground state of a bosonic n-partite system described by the Hamiltonian $\hat{H} = \frac{1}{2}(\sum_i \hat{p}_i^2 + \sum_{i,j} \hat{q}_i V_{ij} \hat{q}_j)$, where $\hat{q}_i$ and $\hat{p}_i$ are the position and momentum operators and the potential matrix is given by $V = \mathscr{C}^2$. We believe that this observation could serve as a starting point for finding useful connections between quantum information theory and quantum statistical physics.

REFERENCES

1. C. Macchiavello and G. M. Palma, Phys. Rev. A **65**, 050301(R) (2002); C. Macchiavello, G. M. Palma and S. Virmani, Phys. Rev. A **69**, 010303(R) (2004); N. J. Cerf, J. Clavareau, C. Macchiavello and J. Roland, Phys. Rev. A **72**, 042330 (2005); V. Giovannetti and S. Mancini, Phys. Rev. A **71**, 062304 (2005); G. Bowen, I. Devetak and S. Mancini, Phys. Rev. A **71**, 034310 (2005); E. Karpov, D. Daems and N. J. Cerf, Phys. Rev. A **74**, 032320 (2006);
2. J. Schäfer, D. Daems, E. Karpov and N. J. Cerf, Phys. Rev. A **80**, 062313 (2009); J. Schäfer, E. Karpov and N. J. Cerf, Proc. of SPIE **7727**, 77270J (2010); J. Schäfer, E. Karpov and N. J. Cerf, Phys. Rev. A **84**, 032318 (2011).
3. O. V. Pilyavets, V. G. Zborovskii and S. Mancini, Phys. Rev. A **77**, 052324 (2008); C. Lupo, O. V. Pilyavets and S. Mancini, New J. Phys. **11**, 063023 (2009); O. V. Pilyavets, C. Lupo and S. Mancini, IEEE Trans. Inf. Theory **58**, 6126 (2012).
4. N. Schuch and J. I. Cirac and M. M. Wolf, *Proc. of Quantum Information and Many Body Quantum Systems*, edited by M. Ericsson and S. Montangero, Vol. 5, (Eidizioni della Normale, Pisa, 2008) pp. 129-142; e-print arXiv:1201.3945
5. G. Adesso and M. Ericsson, Phys. Rev. A **74**, 030305(R) (2006).
6. J. Schäfer, E. Karpov and N. J. Cerf, Phys. Rev. A **85**, 012322 (2012).
7. H. Vahlbruch *et. al.*, Phys. Rev. Lett **100**, 033602 (2008); M. Mehmet *et. al.*, Opt. Express **19**, 25763 (2011).