Symmetry-Driven Thermalization via Finite de Finetti Theorems

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Thermal behavior in subsystems of closed quantum systems is commonly attributed to dynamical chaos, quantum ergodicity, canonical typicality, or the eigenstate thermalization hypothesis, suggesting a fundamentally statistical origin of thermalization. Here, we propose a potential alternative mechanism in which thermal structures emerge deterministically from symmetry considerations alone, without recourse to statistical arguments. We prove a finite de Finetti–type theorem for quantum states invariant under energy-preserving unitaries, establishing that the reduced marginals of any such invariant N-qudit state are close (both in trace distance and relative entropy) to convex mixtures of thermal product states, with explicit error bounds vanishing as $N \to \infty$. We further present an example of Lindblad dynamics whose longtime limit is invariant under energy-preserving unitaries, providing a dynamical realization of the desired symmetry class. These results imply that invariance under energy-preserving unitaries suffices as a sole fundamental, deterministic principle to enforce thermal structures.

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Introduction—Experiments across a wide range of physical situations have explored thermalization in isolated quantum systems, often demonstrating robust local thermal behavior but also exhibiting sometimes striking departures from it in integrable regimes [1–4]. While many systems exhibit relaxation of small subsystems toward thermal (or generalized thermal) states despite a globally unitary, energy-conserving dynamics, others may fail to do so because of strong dynamical constraints. These observations have motivated a large body of theoretical work aimed at clarifying how thermal behavior can emerge, or be obstructed, under reversible microscopic laws [5–20]. Taken together, these approaches can be understood as addressing three interconnected fundamental problems: (P1) identifying the equilibrium thermal states; (P2) explaining how reduced states of subsystems may possess the thermal structure identified in (P1); and (P3) finding physical scenarios in which the microscopic dynamics gives rise to the behavior of (P2), thereby leading to thermalization. For example, contributions addressing (P1) include approaches based on static or variational characterizations of equilibrium states [5–8]. The body of work under (P2) notably includes canonical typicality and the eigenstate thermalization hypothesis [9–13]. Results falling under (P3) focus on equilibration and relaxation under unitary time evolution [4,14,16,17].

The central objective of this work is focused on (P2). Starting from the observation that all current approaches are intrinsically probabilistic, we question whether this is fundamentally unavoidable. Canonical typicality relates thermal behavior to typical properties of states within an energy shell, whereas the eigenstate thermalization hypothesis attributes it to generic features of energy eigenstates and the associated energy spectrum. In both cases, thermalization is inferred from typicality, which is a probabilistic or statistical notion, rather than from symmetry principles or structural constraints on the allowed unitary dynamics. Accordingly, these statistical approaches are formulated without explicit reference to restrictions such as integrability, conserved quantities, or symmetry-imposed selection rules [18,19].

Here, we investigate whether the constraints imposed on the admissible dynamics can serve as a starting point, thereby circumventing the need for a statistical model. In the considered scenario, energy conservation provides a minimal and physically natural restriction, confining the evolution to unitaries that commute with the Hamiltonian. Because this constraint does not single out a unique unitary, and because resolving the detailed transformation would, in practice, require precise microscopic control, we instead describe the dynamics at an effective level by averaging over all energy-preserving unitaries, i.e., by twirling with

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respect to the group of energy-preserving unitaries (EPU). The relevant states are then those invariant under this group.

Guided by this operational viewpoint, we characterize the states of an arbitrary quantum system that are invariant under all EPUs. For any such EPU-invariant state, we prove that the reduced state of a fixed-size subsystem is close (both in trace distance and relative entropy) to a convex mixture of thermal product states, with explicit finite-size bounds. Furthermore, if the total energy distribution is sufficiently narrow, this mixture collapses to an effective single Gibbs state, hence the subsystem thermalizes. Energy-conservation symmetry therefore fixes the local structure of EPU-invariant states and enforces local thermal behavior.

The connection between global symmetry considerations and thermal marginals is illustrated schematically in Fig. 1 for a subsystem of three out of eight qutrits. This mechanism provides a structural explanation of (P2) in that it yields a symmetry-based deterministic account of thermalization at the level of reduced states, without recourse to randomness, chaotic dynamics, or assumptions about typicality or generic spectral properties. Although this constitutes a purely static model for the existence of thermal states in subsystems (P2), we also describe a dynamical scenario in which EPU invariance naturally emerges in the longtime limit, thereby providing a possible route toward thermalization (P3).

Formally, our main result is a de Finetti–type theorem for finite-dimensional quantum states that are invariant under all EPUs. This result is inspired by a related de Finetti theorem for continuous-variable systems [21], where invariance under orthogonal symplectic transformations in phase space (i.e., particle-number preserving Gaussian operations in state space) was shown to lead to convex

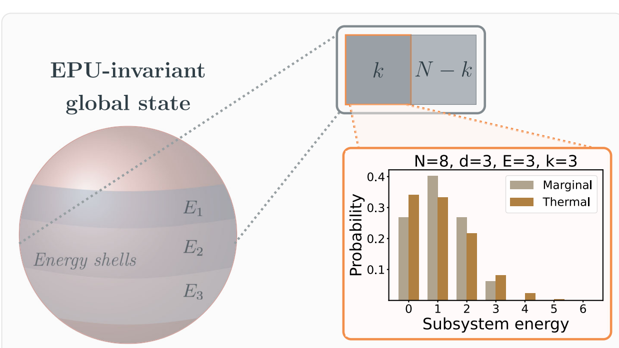

FIG. 1. Symmetry-driven mechanism for the origin of thermalization. The schematic depicts the finite-dimensional de Finetti theorem established here, showing that invariance under EPUs constrains the k-qudit subsystems of an N-qudit system to exhibit thermal reduced states (with a nearly constant temperature if the total energy is narrowly distributed). The inset shows the example of a marginal energy distribution for qutrits ($d=3$) with equidistant energy levels in the case $N=8$, $k=3$, and $E=3$.

mixtures of Gaussian thermal states. In the present setting, our theorem yields a quantitative characterization of reduced states at finite system size and even remains valid under approximate invariance, with deviations being controlled by a suitable measure of asymmetry. At the heart of our approach, we make use of the method of types [22,23], a powerful tool of large deviation theory which constitutes a refinement of the asymptotic equipartition theorem. We note that a special case of thermalization in qubit systems with translation invariance has been analyzed in Ref. [24], which can be understood as exhibiting a finite de Finetti–type structure.

Mathematical preliminaries—Consider N qudits (d-level quantum systems), each with a Hamiltonian h, and denote as $\mathcal{D}_N$ the set of all density matrices of N qudits. Let the eigenvalues (assumed nondegenerate and equidistant) of h be given by E_x, with the corresponding eigenstates $\{|x\rangle\}$ for $x\in\mathcal{X}\coloneqq\{0,\ldots,d-1\}$. Let the total Hamiltonian of the N qudits be $H=\sum_{k=1}^{N}h^{(k)}$. The total energy eigenbasis is therefore $\{|\boldsymbol{x}\rangle\coloneqq|x_1,\ldots,x_N\rangle\}_{\boldsymbol{x}\in\{0,\ldots,d-1\}^N}$. A unitary U acting on the full space is energy preserving if $[U,H]=0$. Let $\mathcal{U}_H$ be the set of $d^N\times d^N$ unitary matrices U satisfying $[U,H]=0$. A state ρ that is invariant under all unitaries in $\mathcal{U}_H$ (called EPU invariant) must be block diagonal in the energy eigenbasis with equal weight on all basis vectors sharing the same total energy. Let $\mathcal{S}_H$ be the set of states that are invariant under all unitaries in $\mathcal{U}_H$; i.e., $\mathcal{S}_H\coloneqq\{\rho\in\mathcal{D}_N|U\rho U^\dagger=\rho,\ \forall\ U\in\mathcal{U}_H\}$. This set is a convex and compact subset of $\mathcal{D}_N$ (see Supplemental Material [25]), so we will focus on its extreme points, namely, the normalized projectors onto the energy eigenspaces of all possible total energies E. A special example of state in $\mathcal{S}_H$ is given by the N-fold tensor product of single qudit thermal states $\tau_\beta=e^{-\beta h}/Z_\beta$, where β is inverse temperature and $Z_\beta=\mathrm{tr}(e^{-\beta h})$ is the partition function. For all $U\in\mathcal{U}_H$, it follows that $U\tau_\beta^{\otimes N}U^\dagger=\tau_\beta^{\otimes N}$.

Method of types—This method allows us to parametrize energy shells in terms of empirical distributions (called types), providing a bridge between combinatorics and thermodynamics. Given a sequence $\boldsymbol{x}=x_1\ldots x_N$ of length N, the *type* $P_{\boldsymbol{x}}$ of sequence $\boldsymbol{x}$ is the relative frequencies of all symbols of $\mathcal{X}$ within sequence $\boldsymbol{x}$ (it can be understood as the empirical probability distribution inferred from sequence $\boldsymbol{x}$; see, e.g., Refs. [22,23]). That is, if n_x is the number of occurrences of symbol x in sequence $\boldsymbol{x}$, then $P_{\boldsymbol{x}}(x)=n_x/N$, with $x\in\mathcal{X}$. Let $\mathcal{P}_N$ denote the set of all possible types P of length-N sequences. Then, for $P\in\mathcal{P}_N$, one defines the *type class* T_P of P as $T_P\coloneqq\{\boldsymbol{x}\in\mathcal{X}^N|P_{\boldsymbol{x}}=P\}$, i.e., the set of all length-N sequences $\boldsymbol{x}$ whose empirical probability distribution matches P. Since the energy of sequence $\boldsymbol{x}$ is given by $\sum_{x=0}^{d-1}n_xE_x$, all sequences $\boldsymbol{x}\in T_P$ have the same total energy given by $E(P)=N\sum_{x=0}^{d-1}E_xP(x)$. The size of the type class T_P is given by a multinomial coefficient, namely,

$$|T_P| = \binom{N}{NP} = \frac{N!}{\prod_{j=0}^{d-1}(NP(j))!}. \quad (1)$$

Since there may be, in general, more than a single type satisfying the same total energy constraint, let us define the set of types with energy $E \in \mathsf{Spec}(H)$ as

$$\mathcal{S}(E) := \{P \in \mathcal{P}_N | E(P) = E\}. \quad (2)$$

Physically, $\mathcal{S}(E)$ accounts for distinct occupation patterns (types) associated with the same total energy E, and the corresponding type classes T_P enumerate the underlying basis vectors at that energy. The total number of strings with energy E is given by $g(E) := \sum_{P \in \mathcal{S}(E)} |T_P|$.

The extreme states $\rho_E^{(N)}$, labeled by total energy E, are the normalized projectors onto the energy eigenspaces, i.e., the subspaces spanned by all basis states $|\boldsymbol{x}\rangle$ whose total energy is E. Equivalently, these are the strings $\boldsymbol{x} \in \mathcal{X}^N$ whose type $P_{\boldsymbol{x}}$ lies in $\mathcal{S}(E)$. Thus,

$$\rho_E^{(N)} = \sum_{P \in \mathcal{S}(E)} \mu_E(P) \times \frac{1}{|T_P|} \sum_{\boldsymbol{x} \in T_P} |\boldsymbol{x}\rangle\langle\boldsymbol{x}|, \quad (3)$$

where $\mu_E(P) = |T_P|/g(E)$ is the fraction of basis states of total energy E that have type P.

Accessing marginals—To express our results, we need to evaluate the partial traces of N-qudit extremal states. Consider a basis sequence $\boldsymbol{y} = y_1 \ldots y_k$ of length k. The type $Q_{\boldsymbol{y}}$ of sequence $\boldsymbol{y}$ is the relative occurrences of the elements of $\mathcal{X}$ within $\boldsymbol{y}$. That is, if m_y is the number of occurrences of symbol $y \in \mathcal{X}$ in the sequence $\boldsymbol{y}$, then $Q_{\boldsymbol{y}}(y) = m_y/k$. For $Q \in \mathcal{P}_k$, define the type class of Q as $T_Q := \{\boldsymbol{y} \in \mathcal{X}^k | Q_{\boldsymbol{y}} = Q\}$, with size $|T_Q| = \binom{k}{kQ}$.

Partial tracing the last $N - k$ qudits of $\rho_E^{(N)}$ as given by Eq. (3), we get the following k-qudit reduced state:

$$\rho_E^{(k)} := \sum_{P \in \mathcal{S}(E)} \mu_E(P) \times \frac{1}{|T_P|} \sum_{\boldsymbol{x} \in T_P} \mathrm{tr}_{N-k}(|\boldsymbol{x}\rangle\langle\boldsymbol{x}|). \quad (4)$$

For each type $P \in \mathcal{S}(E)$, we may express

$$\frac{1}{|T_P|} \sum_{\boldsymbol{x} \in T_P} \mathrm{tr}_{N-k}(|\boldsymbol{x}\rangle\langle\boldsymbol{x}|) = \sum_{\substack{Q \in \mathcal{P}_k \text{ such that} \\ kQ(j) \le NP(j),\ \forall\, j \in \mathcal{X}}} \frac{\prod_{j=0}^{d-1} \binom{NP(j)}{kQ(j)}}{\binom{N}{k}} \times \frac{1}{|T_Q|} \sum_{\boldsymbol{y} \in T_Q} (|\boldsymbol{y}\rangle\langle\boldsymbol{y}|). \quad (5)$$

Defining the multivariate hypergeometric distribution $\mathsf{Hyp}_P^{(N,k)}(Q) = \{[\prod_{j=0}^{d-1} \binom{NP(j)}{kQ(j)}]/[\binom{N}{k}]\}$ and defining the normalized projector onto type-Q length-k sequences as $\hat{\Pi}_Q := (1/[|T_Q|]) \sum_{\boldsymbol{y} \in T_Q} (|\boldsymbol{y}\rangle\langle\boldsymbol{y}|)$, we have

$$\rho_E^{(k)} = \sum_{Q \in \mathcal{P}_k} \left(\sum_{P \in \mathcal{S}(E)} \mu_E(P) \mathsf{Hyp}_P^{(N,k)}(Q) \right) \hat{\Pi}_Q. \quad (6)$$

Note that the above condition $kQ(j) \le NP(j)$ in the sum over $Q \in \mathcal{P}_k$ may be dropped [36].

We need to compare $\rho_E^{(k)}$ to the kth product of the one-qudit energy-diagonal state $\eta_P := \sum_{x=0}^{d-1} P(x)|x\rangle\langle x|$, for any type $P \in \mathcal{S}(E)$. It is easy to see that

$$\eta_P^{\otimes k} = \sum_{Q \in \mathcal{P}_k} \mathsf{Mult}_P^{(N,k)}(Q) \hat{\Pi}_Q, \quad (7)$$

where we have defined the multinomial distribution as $\mathsf{Mult}_P^{(N,k)}(Q) = \binom{k}{kQ} \prod_{i=0}^{d-1} P(i)^{kQ(i)}$. The fact that $\eta_P^{\otimes k}$ depends on N comes from $P \in \mathcal{S}(E)$. Indeed, η_P has the same energy E/N for all $P \in \mathcal{S}(E)$.

Main result—We are now ready to prove a finite de Finetti theorem for EPU-invariant states, which implies that thermal structures emerge deterministically in subsystems of an EPU-invariant state.

Theorem 1—Let $\rho^{(N)}$ be an EPU-invariant N-qudit state and $\tau_\beta^{\otimes k}$ be the thermal (Gibbs) state at inverse temperature β for k noninteracting qudits. Then, for any k-qudit marginal of $\rho^{(N)}$, there exists a mixture of k-qudit thermal states $\{\tau_\beta^{\otimes k}\}_\beta$ such that, for large N and $k \ll \sqrt{N}$, we have

$$\left\| \mathrm{tr}_{N-k}(\rho^{(N)}) - \int \mu(\mathrm{d}\beta) \tau_\beta^{\otimes k} \right\|_1 \le \frac{k(dk + 7d - 2k + 2)}{2N} + O(N^{-3/2 + c\delta}), \quad (8)$$

where $\mu(\mathrm{d}\beta)$ is a valid probability measure on the set $\{\beta\}$ of inverse temperatures, $c > 0$ is some d-dependent constant, and $\delta \in (0, 1/2c)$.

Proof—We may write $\rho^{(N)} = \sum_{E \in \mathsf{Spec}(H)} c_E \rho_E^{(N)}$, i.e., a convex mixture of extreme states in $\mathcal{S}_H$, with the assumption that $E \neq NE_0$ and NE_{d-1} [37]. Let $\mu(\mathrm{d}\beta) := \sum_{E \in \mathsf{Spec}(H)} c_E \delta(\beta - \beta_{E/N}) \mathrm{d}\beta$ be a probability measure, where $\beta_{E/N}$ is the inverse temperature obtained by solving $\sum_{i=0}^{d-1} E_i e^{-\beta_{E/N} E_i} = (E/N) \sum_{i=0}^{d-1} e^{-\beta_{E/N} E_i}$. Then,

$$\begin{aligned} \left\| \mathrm{tr}_{N-k}(\rho^{(N)}) - \int_\beta \mu(\mathrm{d}\beta) \tau_\beta^{\otimes k} \right\|_1 &= \left\| \sum_{E \in \mathsf{Spec}(H)} c_E (\mathrm{tr}_{N-k}(\rho_E^{(N)}) - \tau(E/N)^{\otimes k}) \right\|_1 \\ &\le \sum_{E \in \mathsf{Spec}(H)} c_E \|\rho_E^{(k)} - \tau(E/N)^{\otimes k}\|_1, \end{aligned} \quad (9)$$

where $\tau(E/N) := \tau_{\beta_{E/N}}$. Further, we use the triangle inequality $\|\rho_E^{(k)} - \tau(E/N)^{\otimes k}\|_1 \le \mathsf{norm}\,1 + \mathsf{norm}\,2$, where $\mathsf{norm}\,1 = \|\rho_E^{(k)} - \sum_{P \in \mathcal{S}(E)} \mu_E(P) \eta_P^{\otimes k}\|_1$ and $\mathsf{norm}\,2 = \|\sum_{P \in \mathcal{S}(E)} \mu_E(P) \eta_P^{\otimes k} - \tau(E/N)^{\otimes k}\|_1$. Using Eqs. (6) and

(7), as well as the fact that $\hat{\Pi}_Q$ are orthogonal normalized projectors, we have

$$\begin{aligned}&\mathsf{norm}\,1\\ &\le \sum_{P\in\mathcal{S}(E)}\mu_E(P)\sum_{Q\in\mathcal{P}_k}|\mathsf{Hyp}_P^{(N,k)}(Q)-\mathsf{Mult}_P^{(N,k)}(Q)|\\ &\le\frac{2kd}{N},\end{aligned}\tag{10}$$

where the second inequality comes from a bound on the total variation distance connecting sampling *with* and *without* replacement. In our notations, it is proven that $\sum_{Q\in\mathcal{P}_k}|\mathsf{Hyp}_P^{(N,k)}(Q)-\mathsf{Mult}_P^{(N,k)}(Q)|\le[(2kd)/N]$ in Ref. [38]. This bound has the optimal scaling in k/N.

Now, we upper bound $\mathsf{norm}\,2$ recalling that $\eta_P=\sum_{x=0}^{d-1}P(x)|x\rangle\langle x|$ and $\tau(E/N)=\sum_{x=0}^{d-1}P_\beta(x)|x\rangle\langle x|$, where $P_\beta(x)=e^{-\beta_{E/N}E_x}/(\sum_{y=0}^{d-1}e^{-\beta_{E/N}E_y})$. We obtain

$$\mathsf{norm}\,2\le\frac{k(dk+3d-2k+2)}{2N}+O(N^{-3/2+c\delta}),\tag{11}$$

see Supplemental Material [25] for the technical proof of this inequality. Combining Eqs. (10) and (11) completes the proof of the theorem. ∎

Interpretation of Theorem 1—This finite de Finetti–type theorem implies that the k-qudit marginals of an EPU-invariant N-qudit state are uniformly well approximated, for large N and $k\ll\sqrt{N}$, *by convex mixtures of thermal product states at different inverse temperatures. In Supplemental Material [25], this convergence is illustrated in the simple case of* $N=100$ *qutrits* ($d=3$) *with equidistant energy levels. Note that the approximation does not, in general, lead to a single Gibbs state. An EPU-invariant state may have support on several energy shells, each with a distinct effective temperature, so that the marginal inherits a mixture weighted by the energy distribution. Equivalently, the subsystem is thermal but at an unknown inverse temperature* β *with* $\mu(\mathrm{d}\beta)$ *expressing the uncertainty about* β. *When the energy distribution is narrow, as for macroscopic systems,* μ *concentrates and the mixture approaches a single Gibbs state, yielding conventional thermal behavior in subsystems. In this sense, the finite de Finetti theorem identifies a symmetry-driven mechanism for the emergence of thermal structures, while retaining quantitative control over finite-size deviations. In particular, for any fixed subsystem size k, the deviation between the reduced state and the corresponding thermal mixture vanishes as the total system size N increases, with a convergence rate scaling as* $1/N$.

Although the proof employs asymptotic techniques such as Stirling's approximation and Laplace's method, all estimates are nonasymptotic and hold uniformly for fixed N, $k\ll\sqrt{N}$, and d. In addition, we strengthen this result in Supplemental Material [25] by proving an analogous de Finetti theorem where the relative entropy is used instead of the trace distance to bound the distance between the k-qudit marginals of an EPU-invariant state and the mixture of k-qudit thermal states. Furthermore, we also consider scenarios where the states satisfy only approximate invariance under EPUs. In those cases, we show that the distance between the k-qudit marginals of an EPU-invariant state and the mixture of k-qudit thermal states is additionally controlled by a specific measure of asymmetry of the total quantum state.

Dynamical emergence of EPU-invariant states—Why should EPU-invariant states arise in physical scenarios at all? To answer this question, we consider a physically motivated class of open-system dynamics that converges to an exactly EPU-invariant state, thereby enabling a direct application of our de Finetti theorem. We take our system of N qudits with Hamiltonian H, and, for each total energy E, we define the projector $\hat{P}_E\coloneqq\sum_{P\in\mathcal{S}(E)}\sum_{\boldsymbol{x}\in T_P}|\boldsymbol{x}\rangle\langle\boldsymbol{x}|$. Note that $\mathrm{tr}(\hat{P}_E)=g(E)$. Let the initial state of the system be given by ρ_0. We consider the dynamics generated by a Lindbladian of the form $\mathcal{L}\coloneqq\mathcal{L}_{\mathrm{block}}+\mathcal{L}_{\mathrm{deph}}$, where

$$\mathcal{L}_{\mathrm{block}}(\rho_0)=\sum_E\gamma_E\left[\frac{\mathrm{tr}(\hat{P}_E\rho_0\hat{P}_E)}{g(E)}\hat{P}_E-\hat{P}_E\rho_0\hat{P}_E\right],\tag{12}$$

$$\mathcal{L}_{\mathrm{deph}}(\rho_0)=\sum_E\lambda_E\left[\hat{P}_E\rho_0\hat{P}_E-\frac{1}{2}\{\hat{P}_E,\rho_0\}\right],\tag{13}$$

with $\lambda_E\ge2\gamma_E>0$. Note that the Hamiltonian H does not affect the longtime behavior, so we only consider the energy-preserving dissipative dynamics that is induced by $\mathcal{L}$. We show in Supplemental Material [25] that such a Lindbladian arises in a collision model, and that solving the resulting dynamics yields

$$\begin{aligned}\rho(t)&\coloneqq e^{t\mathcal{L}}(\rho_0)\\ &=\sum_E\left[(1-\varepsilon_E(t))\hat{P}_E\rho_0\hat{P}_E+\varepsilon_E(t)\mathrm{tr}(\hat{P}_E\rho_0)\frac{\hat{P}_E}{g(E)}\right]\\ &\quad+\sum_{E\neq E'}e^{-\frac{1}{2}(\lambda_E+\lambda_{E'})t}\hat{P}_E\rho_0\hat{P}_{E'},\end{aligned}\tag{14}$$

where $\varepsilon_E(t)=1-e^{-t\gamma_E}$. As $t\to\infty$, $\varepsilon_E(t)\to1$ and $e^{-\frac{1}{2}(\lambda_E+\lambda_{E'})t}\to0$, yielding the limiting state $\lim_{t\to\infty}\rho(t)=\sum_E\mathrm{tr}(\hat{P}_E\rho_0)\{[\hat{P}_E]/[g(E)]\}\eqqcolon\mathcal{T}_{\mathrm{EPU}}(\rho_0)$. Thus, the dissipative dynamics converges exponentially to the EPU twirl of the initial state, which is the unique EPU-invariant stationary state compatible with the initial energy distribution.

Let $\Delta\rho_0=\rho_0-\mathcal{T}_{\mathrm{EPU}}(\rho_0)$. Since $e^{t\mathcal{L}}(\mathcal{T}_{\mathrm{EPU}}(\rho_0))=\mathcal{T}_{\mathrm{EPU}}(\rho_0)$, we have $e^{t\mathcal{L}}(\Delta\rho_0)=\rho(t)-\mathcal{T}_{\mathrm{EPU}}(\rho_0)$. Thus, $||e^{t\mathcal{L}}(\rho_0)-\mathcal{T}_{\mathrm{EPU}}(\rho_0)||_1=||e^{t\mathcal{L}}(\Delta\rho_0)||_1$. Then, we show in Supplemental Material [25] that $||e^{t\mathcal{L}}(\rho_0)-\mathcal{T}_{\mathrm{EPU}}(\rho_0)||_1\leq 2e^{-t\Gamma}||\Delta\rho_0||_1\leq 4e^{-t\Gamma}$, where $\Gamma \coloneqq \min_E\{\gamma_E, \lambda_E\}$ and $||\Delta\rho_0||_1 \leq 2$. Choosing $t\geq \Gamma^{-1}\log(4/\epsilon)$ implies that $||e^{t\mathcal{L}}(\rho_0)-\mathcal{T}_{\mathrm{EPU}}(\rho_0)||_1\leq \epsilon$. This means that any state approaches its EPU-twirl under the above Lindbladian dynamics on a timescale $O(\Gamma^{-1}\log(1/\epsilon))$. Therefore, as a consequence of Theorem 1, its marginals thermalize on a timescale $O(\Gamma^{-1}\log(1/\epsilon))$.

Conclusion—We have established a symmetry-based, deterministic mechanism for the emergence of thermal behaviors in quantum subsystems. By proving finite de Finetti–type theorems for states invariant under energy-preserving unitaries, we showed that thermal marginals follow directly from symmetry, with explicit finite-size and dimension-dependent bounds. This yields a nonstatistical account of thermalization that does not rely on chaos, randomness, or ensemble sampling, and hence contrasts with established frameworks such as the eigenstate thermalization hypothesis or canonical typicality. We further showed that this symmetry-based route to thermalization can arise as long-time limits of physically motivated Lindbladian dynamics.

Our proof uses the noninteracting structure $H=\sum_k h^{(k)}$, which allows the energy shells to be identified with type classes. With interactions, two cases arise according to whether the local terms commute. If they do, then, e.g., for nearest-neighbor couplings, shells become classes of second-order types and remain exponentially degenerate [39]. A de Finetti theorem may then be expected [40], with the thermal product states replaced by the reduction of the global Gibbs state. If they do not, the degeneracies are usually lifted, and the symmetry itself would have to be coarse grained to restore them [24]. It will be very interesting to explore these directions.

Finally, since diagnosing thermalization directly from dynamics is difficult [41], it is natural to seek simpler criteria. Noting that symmetry can often be tested efficiently [42,43], the symmetry-based approach developed here suggests that thermal structures could be inferred from the observed EPU invariance. More broadly, our analysis points to symmetry as a potential operational witness of thermality in constrained quantum systems.

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[1] T. Kinoshita, T. Wenger, and D. S. Weiss, A quantum Newton's cradle, Nature (London) **440**, 900 (2006).
[2] T. Langen, S. Erne, R. Geiger, B. Rauer, T. Schweigler, M. Kuhnert, W. Rohringer, I. E. Mazets, T. Gasenzer, and J. Schmiedmayer, Experimental observation of a generalized Gibbs ensemble, Science **348**, 207 (2015).
[3] T. Langen, R. Geiger, and J. Schmiedmayer, Ultracold atoms out of equilibrium, Annu. Rev. Condens. Matter Phys. **6**, 201 (2015).
[4] A. M. Kaufman, M. E. Tai, A. Lukin, M. Rispoli, R. Schittko, P. M. Preiss, and M. Greiner, Quantum thermalization through entanglement in an isolated many-body system, Science **353**, 794 (2016).
[5] E. T. Jaynes, Information theory and statistical mechanics, Phys. Rev. **106**, 620 (1957).
[6] E. T. Jaynes, Information theory and statistical mechanics. II, Phys. Rev. **108**, 171 (1957).
[7] W. Pusz and S. L. Woronowicz, Passive states and KMS states for general quantum systems, Commun. Math. Phys. **58**, 273 (1978).
[8] A. Lenard, Thermodynamical proof of the Gibbs formula for elementary quantum systems, J. Stat. Phys. **19**, 575 (1978).
[9] S. Popescu, A. J. Short, and A. Winter, Entanglement and the foundations of statistical mechanics, Nat. Phys. **2**, 754 (2006).
[10] S. Goldstein, J. L. Lebowitz, R. Tumulka, and N. Zanghì, Canonical typicality, Phys. Rev. Lett. **96**, 050403 (2006).
[11] J. M. Deutsch, Quantum statistical mechanics in a closed system, Phys. Rev. A **43**, 2046 (1991).
[12] M. Srednicki, Chaos and quantum thermalization, Phys. Rev. E **50**, 888 (1994).
[13] M. Rigol, V. Dunjko, and M. Olshanii, Thermalization and its mechanism for generic isolated quantum systems, Nature (London) **452**, 854 (2008).
[14] P. Reimann, Foundation of statistical mechanics under experimentally realistic conditions, Phys. Rev. Lett. **101**, 190403 (2008).
[15] M. Rigol, Breakdown of thermalization in finite one-dimensional systems, Phys. Rev. Lett. **103**, 100403 (2009).
[16] N. Linden, S. Popescu, A. J. Short, and A. Winter, Quantum mechanical evolution towards thermal equilibrium, Phys. Rev. E **79**, 061103 (2009).
[17] A. J. Short and T. C. Farrelly, Quantum equilibration in finite time, New J. Phys. **14**, 013063 (2012).
[18] L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, From quantum chaos and Eigenstate thermalization to statistical mechanics and thermodynamics, Adv. Phys. **65**, 239 (2016).
[19] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Colloquium: Many-body localization, thermalization, and entanglement, Rev. Mod. Phys. **91**, 021001 (2019).
[20] T. Mori, T. N. Ikeda, E. Kaminishi, and M. Ueda, Thermalization and prethermalization in isolated quantum systems: A theoretical overview, J. Phys. B **51**, 112001 (2018).

[21] A. Leverrier and N. J. Cerf, Quantum de Finetti theorem in phase-space representation, Phys. Rev. A **80**, 010102(R) (2009).

[22] T. M. Cover and J. A. Thomas, *Elements of Information Theory* 2nd ed. (Wiley-Interscience, New York, 2006).

[23] I. Csiszár and J. Körner, *Information Theory: Coding Theorems for Discrete Memoryless Systems* 2nd ed. (Cambridge University Press, Cambridge, England, 2011).

[24] M. P. Müller, E. Adlam, L. Masanes, and N. Wiebe, Thermalization and canonical typicality in translation-invariant quantum lattice systems, Commun. Math. Phys. **340**, 499 (2015).

[25] See Supplemental Material at http://link.aps.org/supplemental/10.1103/df89-tdks for detailed proofs, which includes Refs. [22,26–35].

[26] H. Robbins, A remark on Stirling's formula, Am. Math. Mon. **62**, 26 (1955).

[27] T. M. Łapiński, Approximations of the sum of states by Laplace's method for a system of particles with a finite number of energy levels and application to limit theorems, Math. Phys. Anal. Geom. **23**, 9 (2020).

[28] O. Johnson, L. Gavalakis, and I. Kontoyiannis, Relative entropy bounds for sampling with and without replacement, J. Appl. Probab. **62**, 1578 (2025).

[29] A. J. Stam, Distance between sampling with and without replacement, Stat. Neerl. **32**, 81 (1978).

[30] F. Borderi, De Finetti methods in quantum information, Ph.D. thesis, Imperial College London, 2022.

[31] L. Gavalakis, O. Johnson, and I. Kontoyiannis, Finite de Finetti bounds in relative entropy, arXiv:2407.12921.

[32] I. Marvian and R. W. Spekkens, Extending Noether's theorem by quantifying the asymmetry of quantum states, Nat. Commun. **5**, 3821 (2014).

[33] M. Lostaglio, D. Jennings, and T. Rudolph, Description of quantum coherence in thermodynamic processes requires constraints beyond free energy, Nat. Commun. **6**, 6383 (2015).

[34] G. Gour and R. W. Spekkens, The resource theory of quantum reference frames: Manipulations and monotones, New J. Phys. **10**, 033023 (2008).

[35] J. F. C. Kingman, *Poisson Processes* (Clarendon Press, Oxford, 1993).

[36] The condition $kQ(j) \leq NP(j)$ for all $j \in \mathcal{X}$ arises from the requirement that the factors $\binom{NP(j)}{kQ(j)}$ in Eq. (5) must be nonzero. When we sum over P, this condition is automatically enforced by the fact that the corresponding term vanishes whenever it is violated, so we may formally drop the condition.

[37] The energies $E = NE_0$ and NE_{d-1} correspond to two trivial extreme states $\rho_E^{(N)}$ that are pure and remain pure after partial trace. Thus, we must exclude them from Theorem 1 and assume $E \notin \{NE_0, NE_{d-1}\}$ in the rest of the proof.

[38] P. Diaconis and D. Freedman, Finite exchangeable sequences, Ann. Probab. **8**, 745 (1980).

[39] I. Csiszár, The method of types [information theory], IEEE Trans. Inf. Theory **44**, 2505 (1998).

[40] A. Zaman, A finite form of De Finetti's theorem for stationary Markov exchangeability, Ann. Probab. **14**, 1418 (1986).

[41] D. Devulapalli, T. C. Mooney, and J. D. Watson, The complexity of thermalization in finite quantum systems, arXiv:2507.00405.

[42] M. L. LaBorde, S. Rethinasamy, and M. M. Wilde, Testing symmetry on quantum computers, Quantum **7**, 1120 (2023).

[43] S. Rethinasamy, M. L. LaBorde, and M. M. Wilde, Quantum computational complexity and symmetry, Can. J. Phys. **103**, 215 (2025).