INFO-H-451 2014-2015

Information and Coding Theory

Exercise Sheet 3

Exercise 1. Data compression. Determine whether the following codes are nonsingular, uniquely decodable or instantaneous:

(a) The code $\{0,0\}$.

(b) The code $\{0,010,01,10\}$.

(c) The code $\{10,00,11,110\}$.

(d) The code $\{0,10,110,111\}$.

Exercise 2. We consider a source $\{x_i, p_i\}$ with $i = 1, \cdots, m$. The symbols x_i (emitted with probabilities p_i) are encoded in sequences using an alphabet of cardinality D, such that the decoding is instantaneous. For $m = 6$ and lengths of the codewords $\{l_i\} = \{1,1,1,2,2,3\}$, find a lower bound for D. Is this code optimal?

Exercise 3. Huffman code. A source emits a random variable X which can take four values with probabilities $(\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10})$.

(a) Construct a binary Huffman code for X.

(b) Construct a ternary Huffman code for X.

(c) Construct a binary Shannon code for X and compare its expected length with the code of (a).

Exercise 4. Huffman code. A source emits a random variable X which can take four values with probabilities $(\frac{1}{3}, \frac{1}{3}, \frac{1}{4}, \frac{1}{12})$.

(a) Construct a binary Huffman code for X.

(b) Construct a binary Shannon code for X and compare it with the code of (a).

Exercise 5. We have a non-balanced coin with probability p to obtain "head" ("1") and probability $1-p$ to obtain "tail" ("0"). Alice flips this coin as many times as needed to obtain "head" for the first time, and would like to communicate to Bob the number of flips k that were needed. A naive method is to send to Bob the sequence of the outcomes of the coin flips encoded in a chain of bits of length k, like $000\cdots01$ (where 0 stands for "tail" and 1 for "head").

(a) What is the expected length of this naive code? Compare it with the entropy of the random variable k. In which case the naive code is optimal? Use

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$$
$$\sum_{i=0}^{\infty} i\, a^i = \frac{a}{(1-a)^2}$$

(b) Alice decides now to encode her random variable k with a Shannon code, with the aim to approach $H(k)$. Compare the expected lengths of the naive code and the Shannon code in the limit $p \to 0$.

The exercises and solutions are available at `http://quic.ulb.ac.be/teaching`