

INFORMATION AND CODING THEORY
Exercise Sheet 3

Exercise 1. Data compression. Determine whether the following codes are nonsingular, uniquely decodable or instantaneous:

- (a) The code $\{0, 0\}$.
- (b) The code $\{0, 010, 01, 10\}$.
- (c) The code $\{10, 00, 11, 110\}$.
- (d) The code $\{0, 10, 110, 111\}$.

Exercise 2. We consider a source $\{x_i, p_i\}$ with $i = 1, \dots, m$. The symbols x_i (emitted with probabilities p_i) are encoded in sequences using an alphabet of cardinality D , such that the decoding is instantaneous. For $m = 6$ and lengths of the codewords $\{l_i\} = \{1, 1, 1, 2, 2, 3\}$, find a lower bound for D . Is this code optimal?

Exercise 3. Huffman code. A source emits a random variable X which can take four values with probabilities $\{\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}\}$.

- (a) Construct a binary Huffman code for X .
- (b) Construct a ternary Huffman code for X .
- (c) Construct a binary Shannon code for X and compare its expected length with the code of (a).

Exercise 4. Huffman code. A source emits a random variable X which can take four values with probabilities $\{\frac{1}{3}, \frac{1}{3}, \frac{1}{4}, \frac{1}{12}\}$.

- (a) Construct a binary Huffman code for X .
- (b) Construct a binary Shannon code for X and compare it with the code of (a).

Exercise 5. We have a non-balanced coin with probability p to obtain “head” (“1”) and probability $1 - p$ to obtain “tail” (“0”). Alice flips this coin as many times as needed to obtain “head” for the first time, and would like to communicate to Bob the number of flips k that were needed. A naive method is to send to Bob the sequence of the outcomes of the coin flips encoded in a chain of bits of length k , like $000\dots 01$ (where 0 stands for “tail” and 1 for “head”).

- (a) What is the expected length of this naive code? Compare it with the entropy of the random variable k . In which case the naive code is optimal? Use

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$$

$$\sum_{i=0}^{\infty} i a^i = \frac{a}{(1-a)^2}$$

- (b) Alice decides now to encode her random variable k with a Shannon code, with the aim to approach $H(k)$. Compare the expected lengths of the naive code and the Shannon code in the limit $p \rightarrow 0$.