

INFORMATION AND CODING THEORY  
Exercise Sheet 6

**Exercise 1.** Show that the minimum Hamming distance of a linear code is equal to  $d$  if and only if all non-zero codewords have at least  $d$  bits equal to 1, and at least one of them has exactly  $d$  bits equal to 1.

**Exercise 2.** Show that a Hamming code corrects up to  $e - 1$  errors and detects (but does not necessarily correct) up to  $e$  errors if and only if all sets of  $2e - 1$  columns of the parity check matrix are linearly independent.

**Exercise 3.** We define a Hamming code with the  $4 \times 6$  parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & h_{1,6} \\ 1 & 1 & 0 & 0 & 0 & h_{2,6} \\ 0 & 1 & 1 & 0 & 0 & h_{3,6} \\ 1 & 0 & 0 & 1 & 0 & h_{4,6} \end{pmatrix}.$$

- (a) If we set  $h_{1,6} = h_{2,6} = h_{3,6} = h_{4,6} = 1$ , determine the list of codewords. What is the number of information and parity bits? How many errors does this code correct?
- (b) Show that the variables  $h_{1,6}, h_{2,6}, h_{3,6}, h_{4,6}$  cannot be chosen in such a way that the code corrects 1-bit errors and detects (without correcting) 2-bit errors.

**Hint:** Use Exercise 2.

**Exercise 4.** We call a  $k \times n$  matrix  $G$  with  $k$  linearly independent rows (codewords) a generator matrix of a binary code  $(n, k)$ . Each of the  $2^k$  codewords can be expressed as linear combinations of the rows of  $G$ .

- (a) Determine the parity check matrix of the code with the generator

$$G_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

What are the detection/correction properties of this code?

- (b) Same as question (a) but for generator matrix

$$G_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}.$$

The exercises and solutions are available at <http://quic.ulb.ac.be/teaching>