INFOH-422 2015-2016

Information and Coding Theory

Exercise Sheet 6

Exercise 1. Show that the minimum Hamming distance of a linear code is equal to d *if and only if* all non-zero codewords have at least d bits equal to 1, and at least one of them has *exactly* d bits equal to 1.

Exercise 2. Show that a Hamming code corrects up to $e-1$ errors and detects (but does not necessarily correct) up to e errors *if and only if* all sets of $2e-1$ columns of the parity check matrix are linearly independent.

Exercise 3. We define a Hamming code with the 4×6 parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & h_{1,6} \\ 1 & 1 & 0 & 0 & 0 & h_{2,6} \\ 0 & 1 & 1 & 0 & 0 & h_{3,6} \\ 1 & 0 & 0 & 1 & 0 & h_{4,6} \end{pmatrix}.$$

(a) If we set $h_{1,6} = h_{2,6} = h_{3,6} = h_{4,6} = 1$, determine the list of codewords. What is the number of information and parity bits? How many errors does this code correct?

(b) Show that the variables $h_{1,6}, h_{2,6}, h_{3,6}, h_{4,6}$ cannot be chosen in such a way that the code corrects 1-bit errors and detects (without correcting) 2-bit errors.

Hint: Use Exercise 2.

Exercise 4. We call a $k \times n$ matrix G with k linearly independent rows (codewords) a generator matrix of a binary code (n, k). Each of the 2^k codewords can be expressed as linear combinations of the rows of G.

(a) Determine the parity check matrix of the code with the generator

$$G_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

What are the detection/correction properties of this code?

(b) Same as question (a) but for generator matrix

$$G_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}.$$

The exercises and solutions are available at `http://quic.ulb.ac.be/teaching`