INFO-H-422		2017-2018
	INFORMATION AND CODING THEORY Exercise Sheet 6	

NOTE: There were errors in the paper sheet distributed in class. This is a corrected version.

Exercise 1. Show that the minimum Hamming distance of a linear binary code is equal to d *if and only if* all non-zero codewords have at least d bits equal to 1, and at least one of them has *exactly* d bits equal to 1.

Exercise 2. Show that a linear binary code corrects up to $e-1$ errors and detects $2e-1$ errors *if and only if* the minimal number of linearly dependent columns of the parity-check matrix is $2e$.

Exercise 3. Consider a linear binary code with the 4×6 parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & h_{1,6} \\ 1 & 1 & 0 & 0 & 0 & h_{2,6} \\ 0 & 1 & 1 & 0 & 0 & h_{3,6} \\ 1 & 0 & 0 & 1 & 0 & h_{4,6} \end{pmatrix}.$$

(a) If we set $h_{1,6} = h_{2,6} = h_{3,6} = h_{4,6} = 1$, determine the list of codewords. What is the number of information and parity bits? How many errors does this code correct?

(b) Show that the variables $h_{1,6}, h_{2,6}, h_{3,6}, h_{4,6}$ cannot be chosen in such a way that the code corrects 1-bit errors and detects (without correcting) 3-bit errors.

Hint: Use Exercise 2.

Exercise 4. The generator matrix of a linear binary (n,k) code is a $k \times n$ matrix G with k linearly independent rows (codewords). Each of the 2^k codewords can be expressed as linear combinations of the rows of G.

(a) Determine the parity check matrix of the code with generator matrix

$$G_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

What are the detection/correction properties of this code?

(b) Same as question (a) but for the generator matrix

$$G_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}.$$

The exercises and solutions are available at `http://quic.ulb.ac.be/teaching`