

## **Centre for Quantum Information and Communication**

<http://quic.ulb.ac.be>

### **Sujets des Mémoires de Fin d'Etudes pour l'année académique 2013-14**

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Thème général : Sciences de l'Information Quantique

Etudiants concernés : Ir. Physique ou Ms. Science Physique (tous les projets)  
Ir. Informatique ou Ms. Science Informatique (projets 2&3)

Pré-requis : Tous les sujets nécessitent des connaissances de base de mécanique quantique, de théorie des probabilités et d'algèbre linéaire. Une connaissance de base de théorie de la complexité est aussi utile pour le projet 2.

Langue : Français ou anglais en fonction de la personne qui supervise le mémoire (par uniformité, tous les sujets sont présentés en anglais ci-dessous)

#### **1. Capacity of Gaussian quantum channels**

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In communication networks, information is necessarily transmitted via physical carriers, such as, for example, the particles mediating the electromagnetic field, i.e., the photons. Hence, Shannon's information theory must ultimately be consistent with the laws of quantum physics. Modern developments in quantum optics enable the manipulation of electromagnetic signals in a quantum regime, i.e., at the level of single photons, and one can use quantum states as signals for transmitting information. Therefore, a very important problem of Shannon's information theory, namely to determine the capacity of communication channels, may also be formulated for quantum channels. In this context, a particular effort has been devoted to Gaussian bosonic channels as they model most common physical links, such as the optical transmission via optical fibers. Although these channels have been intensively studied, several important problems have remained open today. The goal of this project is to investigate one particular such problem, which can be reduced to the energy-constrained minimization of the output entropy of a lossy channel that couples the input state with a squeezed vacuum state. Proving that the

optimal input state is Gaussian, in analogy to the situation in Shannon's information theory, would bring a big progress in the determination of the capacity of phase-sensitive bosonic channels. The idea is to seek a possible analytical treatment of the optimization problem, guided by a numerical analysis.

## **2. Optimality of quantum algorithms for the Boolean hidden shift problem**

J. Roland (jroland@ulb.ac.be)

Hidden shift problems belong to an important class of problems where quantum computers are known to provide an exponential speedup compared to classical computers. The description of the problem is however very simple : given black-box access to a shifted version  $f(x+s)$  of a function  $f(x)$ , find the hidden shift  $s$ . Depending on the domain and range of the function  $f$ , different versions of the hidden shift problem may be defined. In particular, the Boolean hidden shift problem considers the case where  $f$  is a Boolean function  $f:\{0,1\}^n\rightarrow\{0,1\}$ . A new quantum algorithm for the Boolean hidden shift problem has recently been proposed by us, and shown to provide an exponential speedup for random Boolean functions. However, the complexity of the algorithm can vary greatly for different functions, for example, if  $f$  is a delta function, the problem reduces to Grover's search problem, where only a quadratic speedup is possible. The goal of this projet is to study the complexity of the Boolean hidden shift problem in terms of the Fourier spectrum of the function, which characterizes the cost of the knwown quantum algorithms for this problem, and therefore study their optimality.

## **3. Mixing time of quantum walks**

J. Roland (jroland@ulb.ac.be)

A quantum walk describes the dynamics of a quantum object on a graph, and is the quantum analogue of a random walk or Markov chain. Random walks lie at the core of many classical algorithms: for example, many search problems can be solved by walking randomly on a graph until a certain marked vertex, corresponding to the solution, is reached. In this case, the relevant property of the random walk is its hitting time, that is, the expected number of steps necessary to reach a given vertex. Another important property is the so-called mixing time, which characterizes the number of steps necessary to approach the stationary distribution of the walk. Indeed, many classical algorithms require to sample from some distribution, and this may be achieved by walking randomly on a graph until a sufficiently random vertex is reached. In analogy to random walks, quantum walks have been used to design quantum algorithms, mostly search algorithms whose complexity is related to the so-called quantum hitting time. This project will study the notion of quantum mixing time, exploring its properties and possible applications.

#### **4. Information-theoretic concepts for quantum processes without causal order**

O. Oreshkov (oreshkov@ulb.ac.be)

Quantum information processes are traditionally expressed in a framework that explicitly includes causal ordering between events. For example, one considers a preparer who sends two entangled systems (e.g. two photons) to two separate observers, each of whom performs a measurement on the system she/he receives. An interesting situation is when the two receivers are spacelike separated, which means that the measurement one performs cannot have a causal influence on the other, while the preparer, of course, can influence the two. This is the standard paradigm where nonlocal quantum correlations can be observed. In a recent work, it was shown by us that one can go beyond this and consider situations where the causal ordering between events is not defined, i.e., we can have a “quantum superposition of causal orders”. This new concept will be studied, and an attempt will be made at finding explicit optical setups where such correlations may be observed.

#### **5. Quantum error correction for classical information**

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As classical computing devices rapidly decrease in size (Moore’s law), they approach the domain where the quantum nature of the information carriers and the noise they experience has to be taken into account. Classical error-correcting codes provide a standard method of protecting classical information from noise, but this method is not necessarily optimal at this level. One can easily construct examples of noise models where encoding classical bits into quantum states can offer a more efficient way of storing classical information reliably. The goal of this project will be to investigate the difference between classical and quantum encoding of classical information in different scenarios and to develop a general theory of quantum error correction for classical information.

#### **6. Adiabatic quantum computing via Markovian dynamics**

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Quantum computers promise to solve certain problems more efficiently than classical computers. The standard model of quantum computation works by applying a sequence of quantum gates (or unitary operations) on a set of qubits. An alternative model, which is equivalent in computational power, is Adiabatic Quantum Computing. In this model, the quantum computer is initially prepared in the ground state of a specific Hamiltonian, and the computation works by slowly turning off this Hamiltonian while at the same time turning on another one whose ground state encodes the solution of the computational problem. If this interpolation is performed

slowly enough, the quantum adiabatic theorem guarantees that the initial ground state will be transformed into the final ground state. Recently, the adiabatic theorem was extended from the case of closed quantum system undergoing Hamiltonian dynamics to the case of open quantum systems undergoing dissipative Markovian dynamics, and it was shown that this more general type of adiabatic dynamics can be used to perform various tasks. This project will explore the possibility to perform adiabatic quantum computation via adiabatic Markovian dynamics

## **7. Is randomness easier to communicate over a quantum channel than information?**

R. Garcia-Patron (raulgarciapatron@gmail.com), N. Cerf (ncerf@ulb.ac.be)

The ability to distribute randomness over a channel appears to be a weaker resource than distributing information, as transmitting a meaningful message gives the capability of distributing randomness, while the opposite is not necessarily true. Nevertheless, Shannon's theory of information shows that the optimal rate (capacity) for sending randomness or information in a classical channel is exactly the same. Interestingly, there is some evidence that this equality (between the capacity of sending randomness and information) is no longer true when considering communication over quantum channels, but no final proof of this separation is known. In this project we propose to investigate a family of quantum channels that could lead to a final proof of this open problem.

## **8. Role of quantum entanglement in the optimal measurement of « squeezed » quantum states of light**

E. Karpov (ekarpov@ulb.ac.be), N. Cerf (ncerf@ulb.ac.be)

One of the most important problems in quantum information theory is the information that can be extracted from the measurement of a quantum system. From both operational and conceptual point of views, quantum measurement is a much more complicated notion than measurement in classical physics. For example, it is known that measuring several two-level quantum systems (called « qubits ») may provide more information if we allow a joint operation rather than an individual measurement of each system followed by a classical statistical analysis of the measurement results. In particular, quantum entanglement (an inherently quantum correlation exhibited by quantum systems that has no classical analogue) plays a role in this context as it allows to increase the information extracted from a measurement even if the measured systems are not initially correlated. The goal of this project is to investigate the measurement of several so-called « squeezed » quantum states of light in order to determine if quantum entanglement can also help in this context. An interesting case to study would be to consider a pair of squeezed states that are either identical or phase conjugated. Squeezed states of light are now accessible in a laboratory, which permits to envision (beyond the scope of this project) an

experimental verification of the effect that would be discovered.

## **9. Coupling N single-photon emitter to a single cavity mode**

E. Karpov (ekarpov@ulb.ac.be), E. Brainis (Edouard.Brainis@ugent.be)

This master thesis will consist in modelling the interaction of several atom-like quantum systems with a single mode of an optical resonator. Assuming that once excited each atom can emit a single photon into the cavity mode, we are interested in understanding what quantum states of light can be generated inside the cavity if the excitation of the atoms can be controlled by external means. The work is related to an experimental project running at the Ghent University and consisting in controlling the emission of a several quantum dots (nanometric semiconductor crystals) coupled to the same microdisk resonator. In addition to correctly modelling the problem and classifying the possible interaction regimes, it will be important to focus on the regime that can actually be achieved experimentally and discuss it in depth. The problem depends on many parameters such as the cavity losses, the radiation lifetime and coherence time of the emitters, and even their location in the cavity. Therefore, the student is expected to develop a step-by-step approach, starting from a toy-model, then improving it by adding more and more parameters and understanding their effect.

This research topics is ideal for who wants to understand the interaction of matter with light at its most fundamental level (quantum optics). It requires a good understanding of quantum mechanics, mathematical skills and the ability to translate experimental constrains into model parameters.