Centre for Quantum Information and Communication
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Sujets des Mémoires de Fin d'Etudes pour l'année académique 2016-17

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Thème général : Sciences de l'Information Quantique

Etudiants concernés : Ir. Physique ou Ms. Science Physique (tous les projets)
Ir. Informatique ou Ms. Science Informatique (projets 5-6)

Pré-requis : Tous les sujets nécessitent des connaissances de base de mécanique quantique, de théorie des probabilités et d'algèbre linéaire.

Langue : Français ou anglais en fonction de la personne qui supervise le mémoire (par souci d'uniformité, tous les sujets sont présentés en anglais ci-dessous)

1. Capacity of Gaussian quantum channels

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In communication networks, information is transmitted by physical carriers, such as the photons mediating the electromagnetic field. Modern developments in quantum optics enable the manipulation of electromagnetic signals in a quantum regime, i.e., at the level of single photons, and one uses quantum states of light as signals for encoding information. Therefore, a central problem of Shannon's information theory, namely to determine the capacity of communication channels, may also be formulated for quantum channels. In this context, a particular effort has been devoted to quantum bosonic Gaussian channels as they model most common physical links, such as information transmission via optical fibers. Although these channels have been intensively studied and some central results on the capacity have recently been obtained by a group of researchers including QuIC scientists, several important problems have remained open today.

The first goal of this project is to investigate one particular problem, which can be reduced to the energy-constrained minimization of the output entropy of a lossy channel that couples the input state with a squeezed vacuum state. Proving that the optimal input state is Gaussian, in analogy to the situation in Shannon's information theory, would bring progress in the determination of the capacity of phase-sensitive bosonic channels (below some energy threshold). The idea is to seek a possible analytical treatment of the optimization problem, guided by a numerical analysis. A second possibility may be to recourse to the « replica method », which is a well-known tool in statistical physics for accessing the partition function of disordered systems. This method has also been proven successul in quantum field theory as a means to calculate the von Neumann entropy of a quantum system that circumvents the need for diagonalizing its density matrix. It relies on finding an analytical expression for $\mathrm{tr}(\rho^n)$ with an integer number n of replicas. Then, by analytical continuation, the variable n is considered real and the entropy is accessed via the derivative of $-\mathrm{tr}(\rho^n)$ with respect to n at $n=1$.

The replica method has first been applied to quantum information problems by QuIC researchers, and the present project aims at pursuing this recent and promising research direction. The replica method gives access to the entropy of the output reduced state of an optical parametric amplifier, which cannot be accessed otherwise because of the infinite dimension of the Fock space. In this project, we aim at applying it to the entropic characterization of other quantum Gaussian channels, in order to solve for instance the above mentioned energy-constrained minimization problem.

2. Entropic photon-number inequalities

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A second project following the same lines as project 1 concerns entropic photon-number inequalities for Gaussian bosonic transformations, in particular for the linear coupling induced by a beam splitter. Entropic power inequalities are an extremely powerful tool in classical (Shannon) information theory, allowing to prove a large variety of interesting result, among them the capacity of multiuser channels relevant in telecommunication and Wi-Fi communication. A quantum analogue, called entropic photon-number inequalities, have been conjectured to hold by a group of researchers at the Massachusetts Institute of Technology in 2004, but its proof has remained elusive. Recent progress has shown that a simplified version of this problem can be mapped into a completely classical problem, where one has to minimize the output entropy of a communication channel for a given input probability distribution, over the ensemble of input photon number distributions with an entropy constraint. In this project, we will study the validity of this conjecture numerically in several special cases, and will investigate the possibility of an analytical proof. This would enable approaching the capacity of Gaussian quantum channels in the more general setting of a single emitter and multiple receivers (also called *broadcasting* scenario) or the dual situation of a single receiver but multiple emitters (also called *multiple-access* scenario).

3. Energy-basis majorization a tool for quantum thermodynamics

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Majorization is a well-known mathematical tool that enables comparing probability distributions in terms of their order/disorder. A probability distribution majorizes another one if and only if it there exists a random permutation that degrades the first distribution into the second one. This tool was generalized to quantum states in the 1990s, and several major applications were found in the subsequent years. For instance, the existence of a specific transformation converting a pure entangled state into another one can be expressed based on a quantum majorization relation. In a recent article, QuIC scientists have defined an energy-basis majorization relation (called Fock-majorization), which is a new pre-order relation between quantum states that connects the concept of order with that of mean energy, and has therefore the potential of becoming a useful tool in the study of the thermodynamics of individual quantum systems (called quantum thermodynamics). In this project, we propose to fully characterize the properties of energy-basis majorization in order to reach a similar level of understanding of this new relation as we have for regular majorization today. In a second step, one may seek for new applications of this concept.

4. Entropic uncertainty relations and separability criteria

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Quantum entanglement is viewed as a central resource in quantum communication and computation. Therefore, a main problem in quantum information theory is to determine whether a quantum state is entangled or not, in which case it is called separable and is in some sense useless (its admits a classical description). While this separability problem is simple to solve for bipartite pure state, it is well known to be generally a hard problem for mixed multipartite states. Necessary conditions for separability can be expressed based on the so-called positive partial transpose (PPT) condition. In the specific case of Gaussian bipartite states, this PPT condition has been fully developed and proven to yield a necessary and even sufficient condition for separability. Recently, this study has also been extended to the case of arbitrary (non-Gaussian) states by QuIC researchers among others, and a new condition has been developed based on the link with uncertainty relations.

This project will further exploit the connection between separability conditions and uncertainty relations. In particular, the goal is to investigate entropic uncertainty relations, in which the uncertainties are measured with Shannon entropies instead of variances, unlike in the usual Heisenberg relations. It is expected that a tighter entropic uncertainty relation (to be proven) will give a new tool in order to derive stronger separability conditions (enabling the detection of weakly entangled states that would be left undetected otherwise). The project will start with a numerical check of the conjectured new relations, followed by an attempt to give a proof. It this works, the consequence on separability conditions will be analysed. If this fails, a related goal will be to find new majorization relations that underpin the entropic uncertainty relation in the case where correlations between the observables are taken into account. Furthermore, it should be explored whether a tighter bound on the entropy difference that results from majorization may give rise to tighter entropic separability criteria or uncertainty relations.

5. Search algorithms by random (or quantum) walks in the presence of multiple solutions

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Random walks or Markov chains describe the dynamics of a memoryless particle moving on a graph and have many applications ranging from statistical physics to computer science. For example, the dynamics of a physical system coupled with a thermal bath can be modeled as a Markov chain where each step corresponds to a transition from one state to another due to the absorption or emission of energy. This process converges to the so-called Gibbs distribution which, for low temperatures, favors low energy states. In computer science, this physical process can be simulated to solve optimization problems by mapping the objective function that has to be minimized to energy levels of a physical system: this is the basic idea behind the famous Metropolis algorithm.

Besides optimization problems, many search problems in computer science can be solved by walking randomly on a graph until a certain marked vertex, corresponding to the solution, is reached. In this case, the relevant property of the random walk is its hitting time, that is, the expected number of steps necessary to reach the marked vertex. In the presence of multiple marked vertices (that is, multiple solutions), the algorithm can easily be adapted by stopping as soon as any marked vertex is reached, leading to a notion of hitting time for sets of marked vertices. One disadvantage of this modified algorithm is that it might not provide a fair sampling of solutions, favoring some easily reachable solutions over others.

The goal of this project will be to study how this random walk search algorithm could be modified to provide a fair sampling of solutions. One possible inspiration for this might be the Metropolis algorithm which, by coupling to a thermal bath, allows to escape local minima and uniformly sample global minima. The interested student could also extend his project to the case of quantum walks, which are the quantum equivalents of random walks, and can be used to solve the same search problems.

6. The information complexity of quantum non-locality

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One of the most intriguing aspects of quantum mechanics is quantum non-locality, which arises when two distant systems are measured. Indeed, it can happen that the measurement outcomes are so strongly correlated that, considering a model of the experiment based on classical physics, only faster-than-light communication could explain the outcomes. The correct explanation can only be given by quantum physics, which implies that if two quantum particles have interacted in the past, they can share a special connection called entanglement, which remains even when the particles are separated, and explain these classically impossible correlations.

Non-locality was first considered as an oddity of quantum mechanics, but has now been found the be an invaluable resource for many informational tasks, in particular in the context of cryptography, such as key distribution or randomness generation. For this reason, a natural question is how to quantify the non-locality of an entangled system, and many approaches have been proposed to answer this question. One approach is based on the model of communication complexity. In theoretical computer science, the goal of communication complexity is to quantify the amount of communication required for two distant parties to compute a function f(x,y), if one of them knows x and the other y. In order to quantify non-locality, one might ask how much faster-than-light communication is required to simulate the correlations between the measurement outcomes of the entangled state, which can be shown to reduce to a problem very similar to the standard problem of computing a function.

Recently, the model of communication complexity led to the idea of information complexity, where instead of counting the number of bits sent back and forth between the parties, we count the information content of the messages. This new model has been intensively studied in the last few years and led to a flurry of results, dramatically improving the understanding of the usual communication complexity model.

The goal of this project will be the study the information complexity of quantum non-locality, starting with the case of a maximally entangled pair of qubits. This is the simplest and also most studied system, in particular its communication complexity has been fully characterized as it is known that 1 bit of communication is necessary and sufficient to simulate it. On the other hand, not much is known about its information complexity, except that it is strictly less than one, which is enough to conclude that the question is not trivial and might therefore lead to interesting results.