

Centre for Quantum Information and Communication

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Mémoires de Fin d'Etudes pour l'année académique 2021-22

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Thème général : Sciences de l'Information Quantique

Etudiants concernés : Ir. Physique ou Ms. Science Physique
Ir. Informatique ou Ms. Science Informatique

Pré-requis : Tous les sujets nécessitent des connaissances de base de mécanique quantique, de théorie des probabilités et d'algèbre linéaire.

Note : Certains sujets sont plus proches de la physique quantique (optique quantique, communication quantique, thermodynamique quantique), d'autres de l'informatique quantique (algorithmes quantiques, complexité quantique) ou d'autres encore concernent les fondements de la physique quantique.

Langue : Français ou anglais en fonction de la personne qui supervise le mémoire (par souci d'uniformité, tous les sujets sont présentés en anglais ci-dessous)

1) Boosted optical Bell measurements and partial time reversal

Supervisor : Nicolas Cerf, co-supervisor : Célia Griffet

In the emerging quantum communication and computation technologies, quantum entanglement is often the element that plays the key role. Together with quantum coherence, it enables enhanced informational tasks or can even unlock some otherwise impossible processes. In order to benefit from quantum entanglement, both entangled sources and entangled measurements must be developed. The paradigm of entangled measurements is the so-called Bell measurement, which discriminates the four maximally-entangled states of two qubits. When turning to the quantum optical platform, where qubits are carried by individual photons, a Bell measurement is commonly realized with an optical beam splitter, which is a linear optical component. However, it has long been known that using linear optical components only, the success probability of such an optical Bell measurement cannot exceed 75%. This fundamental limitation motivates the development of « boosted » Bell measurements, which may reach a higher success probability at the expense of using non-linear optics components (hence, making the practical realization more complex). In this Ms. thesis, we intend to explore this direction, which has only been studied partly in the literature. It is recognized that non-Gaussian components may enhance the efficiency, but they are quite complex to implement (and their implementation is most often probabilistic). Thus, we will rather turn to Gaussian components beyond linear optics, especially so-called optical squeezers. We will analyze the few schemes that are known to boost Bell measurements and attempt at improving them. We will also investigate this problem by exploiting a duality (under partial time reversal) between passive and active transformations that has recently been discovered by QuIC researchers. It should enable us to make use of a new language to address this problem. As a second possible line of research, we intend to analyze the dual of the (standard or boosted) Bell measurement setup, where the beam splitter would be replaced by a two-mode squeezer.

2) Majorization relations and quantum entanglement

Supervisor : Nicolas Cerf, co-supervisor : Matthieu Arnhem

The theory of majorization provides a way of comparing probability distributions in terms of disorder. The condition that P majorizes Q means that P is more ordered than Q , in the sense that there exists a random permutation (i.e., a bistochastic matrix) that transforms P into Q , hence P has a lower entropy than Q . While it is a powerful tool in statistics, economy, and various fields based on probabilities, majorization theory also turns out to have interesting applications in quantum information sciences, for example in relation with quantum entanglement or quantum channel capacity. In the special case of bipartite pure states ψ and φ , it gives a necessary and sufficient condition on whether state ψ can be converted into state φ via local operations and classical communication (LOCC). The connection with entanglement lies in that the class of LOCC operations have the property that

they cannot increase the entanglement. More precisely, the vectors of coefficients of the Schmidt decomposition of states ψ and φ form two probability vectors P and Q , and the condition that P majorizes Q implies the existence of a LOCC operation from φ to ψ . The objective of this Ms thesis is to investigate whether this powerful application of majorization theory could be extended to mixed states (statistical mixtures of pure states). This would be significant as mixed states are ubiquitous in realistic conditions. The starting point will be the so-called Hilbert-Schmidt operator decomposition of an arbitrary bipartite mixed state, which comes again with a vector of Schmidt coefficients. The situation is more involved here as these coefficients do not form a probability distribution for mixed states. To overcome this problem, the idea is to exploit the concept of weak majorization and examine whether it could be associated with the entanglement convertibility between two mixed states. This line of research is supported by the connection, recently studied at QuIC, between the vector of Schmidt coefficients of a mixed state and its entanglement in terms of the computable cross-norm criterion.

3) Photonic interference in linear optical circuits

Supervisor: Nicolas Cerf, co-supervisor : Leonardo Novo

Photonic quantum interference is a key resource for implementing future quantum technologies with photonic integrated devices, and in particular for so-called « boson sampling » devices. For this reason, there has recently been a strong interest in genuine multiphoton multimode quantum interference effects, going beyond the celebrated Hong-Ou-Mandel effect (a two-photon interferometric suppression effect that is due to the quantum indistinguishability of two photons impinging on a 50:50 beam splitter: the trajectory where they are both reflected interferes destructively with the trajectory where they both cross the beam splitter). In interferometers with certain symmetries, multiphoton quantum interferences effectively suppress certain sets of detection events, which, mathematically, can be related to properties of the permanent of a matrix describing the interferometer. (NB : The permanent of a matrix is similar to its determinant but with all negative terms being turned into positive terms. It appears here because we deal with photons, hence bosons.) The striking property of permanents lies in that its computation is a $\#P$ -hard problem (this means that, in contrast with determinants, there is no polynomial algorithm for computing the permanent of an arbitrary matrix). In this Ms thesis, we intend to focus on such quantum suppression effects in unbiased interferometers (sometimes called N -splitters), which can be viewed as a N -mode version of a 50:50 beam splitter. Deriving the probability distribution of all patterns at the output of an interferometer is hard, but we will in particular attempt at deriving simplified results on the total photon-number distribution pertaining to some subset of the output modes. As a second line of research, we will analyze the effect of partial indistinguishability of the photons impinging on the interferometer, interpolating between fully distinguishable photons (occupying, for example, non-overlapping time modes) and fully indistinguishable photons (when they all occupy the same time mode). Mathematically, the objective would be to cast this as a « resource theory »

by viewing any fully-distinguishable state as « free » and hence any other state as a « resource » for quantum interferometry. A third possible line of research would be to build on a set of recurrence equations describing multimode multiphoton interferences that have recently been found at QuIC. As these equations combine the permanents and determinants of specific matrices, the objective would be to provide a physical model combining bosonic and fermionic quantum interferences.

4) Energy requirements for measurements in quantum thermodynamics

Supervisor: Nicolas Cerf, co-supervisor : Hamed Mohammady

Thermodynamics is a macroscopic theory applicable in the limit where the number of particles and volume tend to infinity. However, with our increasing ability to control and manipulate microscopic systems, such as the realization of molecular motors and nanomachines, the scope of applicability of thermodynamics is starting to stretch beyond the macroscopic region. One of the key goals of the thermodynamics of small systems – a new field called quantum thermodynamics – is to explore the new features that are brought by the quantum behavior of such small systems. This Ms thesis will be focused on quantum measurements, which are physically implemented by the interaction between the object system to be measured and a measurement apparatus. Quantum measurements do most often incur an energy cost in order to be performed. On the one hand, the physical interaction between the object and apparatus will generally result in an exchange of energy. On the other hand, if we wish to use the same apparatus to repeatedly measure several object systems, we must « erase » the record of the measurement outcomes stored in the apparatus degrees of freedom, which implies a non-negative energy cost that is bounded by Landauer's limit. The objective of this Ms thesis is to pursue a recent research project at QuIC, which aims at investigating the fundamental energy requirements of quantum measurements. The possible research directions that can be taken include: *(i)* investigate what measurements can be implemented with an energy-conserving interaction between the object and apparatus; *(ii)* investigate what measurements can be implemented that incur a zero Landauer erasure cost; *(iii)* combine *(i)* and *(ii)* to determine what measurements can be implemented by a fully « thermodynamically free » measurement process.

5) The computational power of quantum approximate optimization algorithms

Supervisor: Jérémie Roland

As witnessed by Google's recent « quantum supremacy » experiment, the field of quantum computing has entered a new era, with the availability of noisy intermediate-scale quantum (NISQ) computing devices. Quantum approximate optimization algorithms (QAOA) constitute a framework to develop practical applications for such NISQ devices. Unfortunately, from a theoretical point of view we still lack a good understanding of the computational power of QAOA. Another promising framework is adiabatic quantum optimization (AQO), and many

applications of this framework have been proposed since it was proposed in the early 2000's. The goal of this Ms Thesis project is to make progress on our understanding of the computational power of QAOA by adapting various algorithms developed in the context of AQO to the QAOA framework, in order to identify applications where QAOA can provide exponential speedups, or on the contrary where it fails to provide such speedups.

6) Search algorithms via quantum walk with optimal check and update complexity

Supervisor: Jérémie Roland

Random walks are a useful technique to design search algorithms, where the general idea is to map the search space to a graph where some vertices are marked, corresponding to the solutions of the search problem, and to look for these vertices by walking randomly on the graph until a marked vertex is hit. In general, such search algorithms rely on two basic operation: an update operation, which corresponds to a single step of random walk, and a check operation, which corresponds to verifying whether the currently occupied vertex is marked or not. Both of these operations have a specific cost, the update and check cost, which can vary depending on the problem, and in order to minimize the time complexity of a search algorithm by random walk, one needs to minimize the number of calls to the update and check operations.

Quantum walks are the quantum analogue of random walks, and they can also be used to design search algorithms. Typically, they can provide a quadratic speed up over their classical analogue, at least in the situation where the check operation is invoked after each step of the walk. If the check cost is high, it might be worth invoking the update operations multiple times before a vertex is checked, hence only applying check operations once in a while. The goal of this Ms thesis project will be to design improved quantum walk search algorithms, with optimal check and update complexity. Before proceeding with quantum algorithms, the same question could be investigated for classical search algorithms based on random walks.

7) Applications of the quantum linear systems algorithm

Supervisor: Jérémie Roland

Algorithms to solve linear systems find applications in a wide range of fields. Quantum algorithms for solving linear systems outperform its classical counterpart and, in some regimes, fare exponentially better. This Ms thesis project would aim at exploring the applications of this quantum algorithm to various problems and also at techniques that help in extracting useful information from its output. One potential application that could be explored is developing improved quantum algorithms for data-fitting.

8) Quantum processes with indefinite causal order on time-delocalised subsystems

Supervisor: Ognyan Oreshkov, co-supervisor : Julian Wechs

In recent years, the investigation of causal relations in quantum theory has attracted a lot of interest. In particular, it has been found that there exist higher-order quantum processes (that is, transformations that themselves act on quantum operations) which are not compatible with a definite causal order. A central – though pending – question in this context is which of these higher-order processes have a practical realisation, and in what physical situations they can occur. It has been found that some processes with indefinite causal order can be realised on so-called time-delocalised subsystems, that is, by applying the operations on quantum subsystems that are not associated with a definite time.

The aim of this Ms thesis project is to identify and study new examples of quantum processes with indefinite causal order on time-delocalised subsystems.

9) New symmetry transformations through post-selection

Supervisor: Ognyan Oreshkov

The concept of symmetry is fundamental for our understanding of the laws of physics. It was recently shown that reconciling the probabilistic laws of quantum theory with the requirement for time-reversal symmetry requires a generalized formulation of quantum theory, which implies the possibility for more general symmetry transformations than those previously believed possible. This Ms thesis will explore the possibility of realizing this new type of symmetry transformations through post-selection.

10) Quantum Causal Models *vis-à-vis* Quantum Measure Theory: their interplay in Bell scenarios

Supervisor: Ognyan Oreshkov, co-supervisor : Ravi Kunjwal

This project will focus on issues of causality in quantum theory and beyond. Broadly, we aim to use the framework of [Quantum Causal Models \(QCM\)](#) – an agent-centric approach to model causality – to understand aspects of Sorkin's [Quantum Measure Theory](#), an approach to quantum-like theories that seeks realist underpinnings for quantum theory in terms of histories. The latter is motivated by a path-integral (or sum-over-histories) approach to quantum gravity, namely, [causal set quantum gravity](#). We will focus on the relation between these two frameworks in the case of Bell scenarios, where the existence of a strongly positive joint quantum measure (SPJQM) puts strong constraints on quantum Bell violations (cf. [Dowker et al.](#)). Dowker et al. were motivated by a search for ‘quantum Bell causality’, an appropriate generalization of local causality in Bell scenarios. Like local causality, quantum Bell causality would implement an *ontological* notion of relativistic causality

that preserves no-signalling (an *operational* consequence of relativistic causality), but unlike local causality, quantum Bell causality would allow for violations of Bell inequalities. Just as requiring local causality is equivalent to requiring the existence of a joint probability distribution constraining classical correlations in Bell scenarios, they hoped that an appropriate notion of quantum Bell causality would be equivalent to the existence of a SPJQM. The framework of QCMs, on the other hand, comes with the quantum Markov condition, which – at first glance – seems to provide a candidate for a ‘quantum Bell causality’ condition in the sense that it generalizes the classical Markov condition, the analog of local causality. We will explore whether this qualitative insight can be turned into a quantitative result, perhaps reformulating the condition of SPJQM in terms of a constraint on process operators, and/or conversely. The consequences of this for realist underpinnings of quantum causal models (QCMs) will also be explored. This Ms thesis project could also provide insight into the challenge of formulating an ‘almost quantum theory’ that realizes the ‘almost quantum’ set of correlations in Bell scenarios (cf. [Boes and Navascues](#)). Another worthwhile direction, moving away from Bell scenarios, is to explore the connection between the lack of third-order interference and contextuality in quantum theory (cf. [Henson](#)).

11) Contextuality in quantum physics

Supervisor: Ognjan Oreshkov, co-supervisor : Ravi Kunjwal

The notion of contextuality in quantum theory arose from foundational considerations starting in the 1960s and, in its modern form, multiple incarnations of this notion have appeared in the quantum foundations and quantum information literature. In this project, we will adopt a [noise-robust notion of contextuality](#), first proposed by [Spekkens](#). The goal of this project will be to look for the role that contextuality plays in quantum physics, broadly understood: in particular, what are the physical phenomena in which contextuality naturally arises and how might one witness it in an experiment? We aim at developing applications of contextuality where its witness is native to the phenomenon of interest and quantitatively useful. Some potential directions include (but are not limited to):

- Contextuality in [quantum computation with magic states](#),
- Contextuality in [quantum thermodynamics](#),
- Contextuality vis-à-vis [anomalous weak values](#),
- Applications of [hypergraph frameworks](#) for noise-robust contextuality to quantum information, and
- Foundational role of contextuality *vis-à-vis* [interpretational issues in quantum theory](#).

The specific course of the Ms thesis project will depend on the individual interest(s) of the student(s).