Centre for Quantum Information and Communication
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Mémoires de Fin d'Etudes pour l'année académique 2022-23

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Thème général : Sciences de l'Information Quantique

Etudiants concernés : Ir. Physique ou Ms. Science Physique
Ir. Informatique ou Ms. Science Informatique

Pré-requis : Tous les sujets nécessitent des connaissances de base de mécanique quantique, de théorie des probabilités et d'algèbre linéaire.

Note : Certains sujets sont plus proches de la physique quantique (optique quantique, communication quantique, thermodynamique quantique), d'autres de l'informatique quantique (algorithmes quantiques, complexité quantique) ou d'autres encore concernent les fondements de la physique quantique.

Langue : Français ou anglais en fonction de la personne qui supervise le mémoire (par souci d'uniformité, tous les sujets sont présentés en anglais ci-dessous)

1) Lattice majorization in the theory of quantum entanglement
Supervisor: Nicolas Cerf, co-supervisor: Célia Griffet

In the emerging fields of quantum computation and communication, the existence of quantum entanglement between quantum bits if often the key element that provides what is called a quantum advantage. Together with quantum coherence and quantum nonlocality, it often enables enhanced informational tasks or it can sometimes even unlock some otherwise classically impossible processes. In order to benefit from this advantage, a theory of quantum entanglement must be developped, aiming at certifying whether a quantum state is separable or entangled, and, in the latter case, aiming at measuring the amount of entanglement. In this context, the concept of majorization is known to play a main role.

The mathematical theory of majorization provides a way of comparing two probability distributions in terms of disorder. The condition that P majorizes Q means that P is more ordered than Q, in the sense that there exists a random permutation (i.e., a bistochastic matrix) that transforms P into Q, hence P has a lower entropy than Q. While it is a powerful tool in statistics, economy, and various fields using probabilities, majorization theory also turns out to have interesting applications in relation with quantum entanglement. In the special case of bipartite pure states ψ and φ, it gives a necessary and sufficient condition on whether the entangled state ψ can be converted into the entangled state φ via local operations and classical communication (LOCC), a class of operations that cannot increase entanglement.

The objective of this Ms thesis is to investigate potential applications of the so-called majorization lattice to entanglement theory. The majorization lattice extends majorization to a collection of probability distributions and can be understood using an analogy with the "least common multiple" and "greatest common divisor". Little is known as of today on its application to quantum states, which motivates this project. A first possibility could be to consider a lattice of bipartite quantum states, while a second possibility could be to the consider the lattice of reduced states starting a multipartite quantum state. The objective would be to derive new interconvertion conditions or new separability conditions.

2) Search algorithms via quantum walk with optimal check and update complexity

Supervisor: Jérémie Roland

Random walks are a useful technique to design search algorithms, where the general idea is to map the search space to a graph where some vertices are marked, corresponding to the solutions of the search problem, and to look for these vertices by walking randomly on the graph until a marked vertex is hit. In general, such search algorithms rely on two basic operation: an update operation, which corresponds to a single step of random walk, and a check operation, which corresponds to verifying whether the currently occupied vertex is marked or not. Both of these operations have a specific cost, the update and check cost, which can vary depending on the problem, and in order to minimize the time complexity of a search algorithm by random walk, one needs to minimize the number of calls to the update and check operations.

Quantum walks are the quantum analogue of random walks, and they can also be used to design search algorithms. Typically, they can provide a quadratic speed up over their classical analogue, at least in the situation where the check operation is invoked after each step of the walk. If the check cost is high, it might be worth invoking the update operations multiple times before a vertex is checked, hence only applying check operations once in a while. The goal of this project will be to design improved quantum walk search algorithms, with optimal check and update complexity. Before proceeding with quantum algorithms, the same question could be investigated for classical search algorithms based on random walks.

3) Quantum linear systems algorithms in continuous time

Supervisor: Jérémie Roland

Algorithms to solve linear systems of equations find applications in a wide range of fields. Quantum algorithms for solving linear systems outperform their classical counterparts and, in some regimes, fare exponentially better. Achieving optimal complexity in terms of both the condition number of the system matrix and the precision of the solution however requires rather complicated constructions. Most of the known algorithms are expressed in the standard gate-based model of quantum computation, where the time evolution is discrete. However, there also exist alternative (and computationally equivalent) models of quantum computation where the system evolves continuously in time under the effect of a Hamiltonian, and such models sometimes lead to simpler algorithms. This project would aim at developing new quantum linear systems algorithms in continuous time, with the goal of achieving optimal complexity with simpler, more intuitive constructions.

4) Quantum general covariance as an extended symmetry principle
Supervisor: Ognyan Oreshkov, co-supervisor: Lin-Qing Chen

General covariance - the invariance of the form of dynamical equations under coordinate transformations - is a basic principle of physics law. Traditionally, the concept of coordinates was the abstraction from classical reference frames; but to study physics at the small scale, we have to take into account that all physical systems are essentially quantum, including the reference frames. Recently, the new development of *quantum reference frames* and their transformations have taught us that both entanglement and localization of events are reference frame-dependent concepts. In this new realm of physics, an *extended symmetry principle* has been postulated. It states that the physical laws shall retain their form under quantum coordinate transformation, which has broadened the notion of covariance. The extended symmetry principle has been shown to be very powerful to give new physical predictions by relating unknown quantum configurations with the known ones by the symmetry transformation defined upon quantum reference frame. It was even conjectured to be a guiding principle for a theory of quantum gravity.

Despite of its promise and importance, the extended symmetry principle has only been justified in a few simple examples - we do not know to which extent it will hold. This is partially because the notion of *quantum coordinate* has been very ambiguously used in the community, and it has not yet converged to a clear definition. In this Master project, we will first compare different versions of *quantum coordinates* in the literature and analyze what might be the best suited notion. Then we will investigate some concrete physical scenarios (for example, the superposition of different configurations of massive particles) to analyze quantum coordinate transformations and falsify or broaden the validity of quantum general covariance. If the research progresses smoothly, based on the examples we have studied, we will propose a precise mathematical definition for general *quantum coordinate transformations* and summarize the validity conditions for q*uantum general covariance* as an extended symmetry principle.

5) Quantum processes with indefinite causal order on time-delocalised subsystems
Supervisor: Ognyan Oreshkov, co-supervisor: Julian Wechs

In recent years, the investigation of causal relations in quantum theory has attracted a lot of interest. In particular, it has been found that there exist higher-order quantum processes (that is, transformations that themselves act on quantum operations) which are not compatible with a definite causal order. A central question in this context is which of these higher-order processes have a practical realisation, and in what physical situations they can occur. It has been found that some processes with indefinite causal order can be realised on so-called time-delocalised subsystems, that is, by applying the operations on quantum subsystems that are not associated with a definite time. The aim of this Master thesis project is to identify and study new examples and possible applications of quantum processes with indefinite causal order on time-delocalised subsystems.

6) Contextuality in quantum physics

Supervisor: Ognyan Oreshkov, co-supervisor: Ravi Kunjwal

The notion of contextuality in quantum theory arose from foundational considerations starting in the 1960s and, in its modern form, multiple incarnations of this notion have appeared in the quantum foundations and quantum information literature. We will adopt a noise-robust notion of contextuality, first proposed by Spekkens. The goal of this project is to look for the role contextuality plays in quantum physics, broadly understood: in particular, what are the physical phenomena in which contextuality naturally arises and how might one witness it in an experiment? We aim to develop applications of contextuality where its witness is native to the phenomenon of interest and quantitatively useful. Some potential directions include (but are not limited to):

- Contextuality in quantum computation with magic states,
- Contextuality in quantum thermodynamics,
- Contextuality vis-à-vis anomalous weak values,
- Applications of hypergraph frameworks for noise-robust contextuality to quantum information, and
- Foundational role of contextuality vis-a-vis interpretational issues in quantum theory.

The specific course of the Master project will depend on the individual interest(s) of the student(s).